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**Essays on Unemployment Volatility**

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# Essays on Unemployment Volatility

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A thesis submitted for the degree of Doctor of Philosophy

University of Bath  
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September 2016

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# Abstract

This thesis analyses different approaches to address the unemployment volatility puzzle. In the first two chapters, we develop two types of search frictions model with efficiency wages. The models can match observed fluctuations in unemployment and job vacancies in the U.S economy. Moreover, the models also capture labour market dynamics well. In the third chapter, we analyse two proposed solutions to the unemployment volatility puzzle: sticky wages and a small ‘hiring surplus’. We investigate a widely used calibration strategy in the literature and argue that it is a key factor in generating large unemployment volatility. In the fourth chapter, we reassess the following arguments on the unemployment volatility puzzle: strategic wage bargaining; large fluctuations in discount rates in the financial market; and endogenous job separations caused by idiosyncratic productivity shocks.

# Overview

In recent decades, the Diamond-Mortensen-Pissarides (DMP) search and matching model has emerged as the workhorse model for the analysis of unemployment and related policy issues. It gives an appealing description of how unemployment arises in equilibrium and what makes it change over time. However, as is widely known, the model is incapable of matching the observed fluctuations in unemployment and job vacancies; this is often referred to as the unemployment volatility puzzle.

In this thesis, we analyse different approaches to address this puzzle. Our first approach involves modelling a different approach to wage formation based on efficiency wage theory. The motivation for doing this is the fact that the wage under Nash bargaining in the standard DMP model is too sensitive to labour productivity, causing job vacancies and unemployment to have almost no response to productivity shocks, contrary to what is observed in the data. In the first chapter, we replace Nash wage bargaining with a Solow-type efficiency wage mechanism, in which the amount of effort expended by workers is determined by the proportional gap between the real wage and a reference wage (see for example, Solow (1979) and Summers (1984)). In this efficiency wage context, the cyclicalities of the optimal wage largely depends on the cyclicalities of the reference wage.

We propose two alternative ways to define the reference wage. One incorporates the idea of ‘social norms’ and ‘fair wages’ in the efficiency wage

literature (see for example, Elster (1989) and Akerlof and Yellen (1988)). The second one is based on search theory; we use the expected discounted value of being unemployed as the reference wage. We show that, in both cases, wages are less responsive to productivity shocks than with Nash bargaining. The economic intuition behind the second approach is especially interesting. Under rational expectations, workers realise that a temporary technology shock only has a limited effect on their outside option which is based on their expected life-time career path. So workers know that any increase in the wage they might earn from alternative employment that results from a positive productivity shock cannot last long since the shock is temporary. This interpretation is similar to the ‘permanent income hypothesis’ in consumption theory, see Friedman (1957).

In the second chapter, we explore the implications of an alternative type of efficiency wage mechanism for unemployment volatility. Following Shapiro and Stiglitz (1984), we assume that effort is binary, corresponding to a worker choosing to shirk or not. In this context, the wage responds to a productivity shock if and only if the job finding rate responds to productivity. If the job finding rate increases after a positive productivity shock, then the relative value of being unemployed to a worker also increases. This reduces the potential loss from being fired if the worker chooses to shirk since it becomes easier to find a new job. As a result, the firm has to raise the wage to increase the threat point faced by workers. Empirical evidence shows that job finding rate is not as volatile as other labour market variables such as labour market tightness. This provides another way to explain the mild cyclicalities of the wage.

The first two chapters show that efficiency wage theory can address the unemployment volatility puzzle when this is incorporated into a dynamic labour market with search frictions. Using standard calibrations, we show that the models in the first two chapters can match the observed value of key labour market indicators in the U.S economy, such as the average unemployment rate and the rates at which unemployed workers find jobs. Moreover, the models can generate the observed business-cycle-frequency fluctuations in unemployment, job vacancies and the vacancy-unemployment ratio in re-

sponse to shocks of a plausible magnitude. Also, the models outperform the standard DMP model in terms of capturing some dynamic features of the U.S labour market, such as the correlations and autocorrelation of key labour market indicators.

There is a large amount of work discussing alternative approaches to address the unemployment volatility puzzle. For example, Hagedorn and Manovskii (2008) controversially raise the value of leisure when they calibrate the standard DMP model; Pissarides (2009) introduces fixed matching costs to the DMP model; Petrosky-Nadeau and Wasmer (2013) embed credit frictions into the DMP model.

In the third chapter, we point out a common channel behind those alternative approaches. This common channel works through a high rate of filling a vacancy. A high rate of filling a vacancy makes any newly posted vacancy due to a positive productivity shock more likely to be filled and makes any reduction of job vacancies due to a negative productivity shock more likely to be transmitted to job losses. This generates larger fluctuations in unemployment. We argue that to achieve a high vacancy-filling rate, those scenarios diminish the firm's benefit from hiring a worker. This leads to low incentives for vacancy creation and causes increased slack in the labour market, which in turn shortens the duration of a vacancy and increases the vacancy-filling rate. We refer this channel as the 'small hiring surplus'. We argue that, in terms of generating unemployment volatility, the 'small hiring surplus' is a complement to another common approach which is often referred as the 'sticky wage'. The 'sticky wage' ensures that job vacancies vary with labour productivity. The 'small hiring surplus' ensures that any variation in job vacancies is more likely to be transmitted to variation in unemployment.

Also in the third chapter, we investigate a widely used calibration strategy in the literature, which is obscured by different time periods used in calibrations (see for example, Hall (2005), Hagedorn and Manovskii (2008) and Hall and Milgrom (2008)). This calibration strategy raises the relative cost of creating a job vacancy. We adjust the calibrations used in different papers to make them all correspond to a monthly frequency. It turns out that a



large vacancy posting cost is associated with a high matching efficiency and high vacancy-filling rate. We further argue that this strategy implies a large unemployment volatility.

The fourth chapter contains a critical review of recent developments in the literature on the unemployment volatility puzzle. Specifically, we reassess the arguments on: strategic wage bargaining; large fluctuations in discount rates in the financial market; and endogenous job separations caused by idiosyncratic productivity shocks.

We argue that the success of models with strategic wage bargaining in addressing the unemployment volatility puzzle is largely due to the assumption a fixed cost of delay in wage negotiations. However a fixed cost of delay may not be a plausible assumption in the search frictions model since the major cost of delay is likely to be the value of the lost production due to the delay of reaching a wage agreement, which is pro-cyclical. We argue that if we assume a pro-cyclical cost of delay, the model with strategic wage bargaining cannot replicate the observed unemployment fluctuations under plausible calibrations of the structural parameters.

There is a rapidly growing literature focusing on the observed co-movement between the job hiring and discount rates in the U.S financial market. One of the most recent papers, written by Hall (2015), suggests that the discount rate in the stock market is a driving force of unemployment fluctuations in the U.S economy. Although we acknowledge this co-movement, we have doubts about the mechanism Hall (2015) proposed. Specifically, we doubt that the discount rate is the main channel that transmits fluctuations in financial markets to job hiring. We show that if Hall's hypothesis were true, then if we endogenize job destruction in the standard DMP model, one implication would be that job destructions decrease with the discount rates in the stock market, which contradicts the observed facts in financial crisis. We also show that Hall (2015)'s results rely heavily on his wage specification.

In the final section, we show that endogenous job destruction can increase the volatility of labour market tightness, especially when reservation productivity is high. Reservation productivity is the lower bound of production below which job separation occurs. High reservation productivity triggers

large flows into unemployment. Large inflows to unemployment slacken the labour market, increasing the outflows from unemployment. The two larger flows lead to a more volatile labour market. We also show that reservation productivity is negatively correlated with aggregate productivity. One implication is that job destruction may not be a key factor for generating unemployment volatility in ‘normal’ periods, but probably is a key factor in recessions.

# Chapter 1

## Search Frictions, Efficiency Wages and Unemployment Fluctuations

### Abstract

This chapter analyses unemployment fluctuations in a model that adds a Solow-type efficiency wage effect to an otherwise standard search frictions model of the labour market. We argue that our model outperforms the standard search frictions model in two key aspects: (i) our model can generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of a plausible magnitude. This is largely because wages are less responsive to unemployment when efficiency wage effects are considered; (ii) our model also captures labour market dynamics well. We also extend the efficiency wage literature by (i) extending the well-known Solow condition to allow for search and matching frictions and labour market flows and (ii) providing a search-theoretic microfounded reference wage.

# 1 Introduction

Explaining unemployment is one of the central problems in economics. The currently dominant approach in the literature is the Diamond-Mortensen-Pissarides (DMP) search and matching model (e.g. Diamond, 1982, Mortensen and Pissarides, 1999, Pissarides, 2000). This provides a simple framework for the analysis of the labour market and associated policy issues (eg, Royal Swedish Academy of Sciences, 2010). However, as widely known, the model is incapable of generating the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of a plausible magnitude, which is often referred as the unemployment volatility puzzle.

The standard DMP model has two main elements. The first is a model of the imperfect matching of workers with jobs, explaining why unemployed workers coexist with unfilled job vacancies. This is an essential part of the model. The second is a model of how the surpluses accruing to workers and firms from a job match are divided between the parties through bargaining over the real wage. In the search frictions model, Nash wage bargaining ensures that the real wage exceeds the marginal rate of substitution between consumption and leisure and thus explains the existence of involuntary unemployment. However, the literature has argued that the wage under Nash bargaining is too sensitive to labour productivity, causing job vacancies and unemployment to have almost no response to productivity shocks.

In this paper, we replace Nash bargaining with efficiency wages and explore this alternative type of search frictions model. There is a small existing literature on this. Malcomson and Mavroeidis (2007) develop an empirical model that incorporates a Shapiro and Stiglitz (1984) no-shirking constraint into a search frictions model; see also Pissarides and Mortensen (1999) and Zaharieva (2010). We take a different approach. We combine search frictions with a simple model of efficiency wages, similar to Solow (1979) and Summers (1984)<sup>1</sup>, in which output depends on technology and the amount of effort expended by workers, which is determined by the proportional gap

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<sup>1</sup>Danthine and Kurmann (2008, 2010) incorporate a Solow-type efficiency wage mechanism into a DSGE model, the paper addresses the observed response of macro-variables to monetary policy shocks.

between the real wage and a reference wage. In a pure efficiency wage context, an individual firm minimizes the cost of effective labour by maintaining an optimal proportional gap between the real wage and the reference wage. When a productivity shock hits the economy, firms adjust the real wage if (i) the reference wage changes with productivity; or (ii) the optimal proportional gap changes with productivity.

We propose two alternative ways to define the reference wage. One incorporates the idea of ‘social norms’ and ‘fair wages’ in the efficiency wage literature (see for example, Elster (1989) and Akerlof and Yellen (1988)). The second one is based on search theory, choosing the expected discounted value of being unemployed as the reference wage. We show that, using standard values for structural parameters, the reference wage in both set ups have little response to productivity, displaying strong stickiness. The stickiness of the reference wage based on social norms and fair wages comes from the psychological hypothesis that workers normally prefer to receive a smooth income flow. The stickiness of the reference wage based on search theory comes from the fact that a temporary productivity shock can hardly have long-lasting effect on both the probability of finding a job and the future wage payment, therefore having a limited effect on a workers’ reservation wage.

We show that efficiency wage theory can explain why wages are less responsive to productivity shocks when this hypothesis is incorporated into a dynamic labour market with search frictions. When a Solow-type efficiency wage model is considered, the cyclicity of the wage depends on the cyclicity of the reference wage of workers. We give two alternative ways to define the reference wage. One is based on the idea of social norms and fair wages. The other one is based on the expected return of outside option. In the second approach, under rational expectations, workers realise that a temporary technology shock only has a limited effect on the expected return of the outside option which is measured based on their expected life-time career path. So unless workers are ‘myopic’, workers know even if they successfully switch to another job which comes with a higher payment when a positive productivity shock is observed, such a relatively high payment

cannot last long since the shock is temporary. As a result, they prefer to stay in the current job to receive a stable income although their employers do not increase the wage much. This microfounded reference wage plays a key role in delivering the little cyclical variation of the wage.

The chapter has four contributions. First, we develop a simple analytical model by adding the efficiency wage effect described above to an otherwise standard search frictions model. We obtain a simple expression for the optimal wage. Wages have similar determinants as in the standard DMP model: income available to unemployed workers, the job vacancy posting-cost, the rate at which job matches are dissolved and the unemployment rate. However the mechanisms behind wage formation are different. In the search frictions model, wages are determined through the sharing of the surplus from a continuing job match between worker and firm. Thus a rise in the cost of posting a vacancy increases the value of a continuing job match and so increases the wage. This increases the cost of labour and leads to lower employment. In the efficiency wage model, by contrast, the firm sets the wage unilaterally. The wage affects productivity as well as the cost of labour and so the firm chooses the wage to minimise the cost of labour per unit of effort. A rise in the cost of filling a vacancy will lead to a reduction in employment and an increase in effort, this latter requiring an increase in the wage. The wage in our model displays much stronger stickiness in terms of its response to productivity shocks than the Nash bargained wage. Therefore we provide an alternative approach to modelling the sticky wage.<sup>2</sup>

Second, using standard values for the structural parameters, we show that our model can match the observed values of key labour market indicators in the U.S economy, such as the average unemployment rate and the rate at which unemployed workers find jobs. Moreover, our model can match the observed business-cycle-frequency fluctuations in unemployment, job vacancies and vacancy-unemployment ratio in response to shocks of a plausible magnitude. Third, we show that, again using standard values for struc-

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<sup>2</sup>A leading alternative in the literature is strategic wage bargaining, see Hall and Milgrom (2008). A shortcoming of strategic wage bargaining is discussed in the fourth chapter of the thesis.

tural parameters, our model outforms the standard DMP model in terms of capturing some dynamic features of the U.S labour market, such as the correlation and autocorrelation of key labour market indicators.

Finally, we also extend results in the existing efficiency wage literature. Our model simplifies to a standard efficiency wage model if job matching is perfect. Wage-setting is characterised by an extension of the Solow (1979) condition to allow for the dynamics of the labour market and the effects of imperfect matching. In the efficiency wage model, the productivity of workers depends on the wage paid to workers relative to a reference wage. The existing literature assumes ad-hoc functions for the reference wage that reflect current wages and unemployment. As discussed earlier, one of the reference wages in our model takes a more search theoretic approach by relating the reference wage to the forward-looking value functions associated with being unemployed. This search-theoretic microfounded reference wage fills the gap between efficiency wage theory and search theory.

Currently, the strategic bargaining approach is the most prominent response to the unemployment volatility puzzle (Mortensen and Nagypal, 2007, Hall and Milgrom, 2008, Christiano et al, 2016). Notwithstanding this, there are issues with the strategic bargaining approach. The approach assumes that the threat point in wage bargaining is delay rather than break-up of the match and that the firm incurs a fixed cost of delay and the worker enjoys leisure in the event of disagreement in a bargaining round. Ljungqvist and Sargent (2016) stress the model's reliance of a large calibrated cost of delay. These threat points are acyclical and so result in the wage being less cyclically sensitive than in simple Nash bargaining. Since there is no production until agreement is reached, it might be argued that the cost of delay to the firm should be seen as pro-cyclical, reflecting lost output, rather than fixed. If so, the threat point becomes more pro-cyclical and so does the resulting wage. Other literature in this field solves the puzzle by diminishing the firms' profits. For example, Hagedorn and Mankovskii (2008) raise the value of utility of leisure; Pissarides (2009) introduces a fixed cost of matching in addition to costs of posting vacancies and Petrosky-Nadeau and Wasmer (2013) introduce credit frictions. In the literature, the particular

weakness of the search frictions model in explaining vacancies has received less attention.

The remainder of the paper is structured as follows. We outline the model in section 2) explaining the objectives and constraints of households and firms and explaining job matching and labour market flows. Then we characterise the model by deriving the optimality conditions for employment and wages. In section 3) we calibrate and simulate the model. Section 4) concludes.

## 2 The Model

### 2.1 Households

There is a continuum of identical individuals on the unit interval. Each individual inelastically supplies one unit of labour in every period and consumes all the income they earn. An individual is either employed and earning a wage  $w$ , or else unemployed and earning real unemployment benefits  $b$ . If unemployed, an individual finds a job with probability  $f_t$ . At the end of each period, existing job matches are exogenously terminated with probability  $\tau_t$ . We assume  $\tau_t = \tau \varepsilon_t^\tau$  where  $\varepsilon_t^\tau$  is a separation shock. Since we assume that all firms are identical, the value of being employed is thus

$$L_t = w_t + \beta E_t[(1 - \tau_t)L_{t+1} + \tau_t U_{t+1}] \quad (1.1)$$

where  $\beta$  is the discount factor. We define that  $\beta = 1/(1 + r)$  where  $r$  is the real discount rate. The value of being unemployed is

$$U_t = b + \beta E_t[f_t L_{t+1} + (1 - f_t)U_{t+1}] \quad (1.2)$$

#### 2.1.1 The Fair Level of Effort

If employed, individuals exert what they regard as the "fair" level of effort. We assume that the fair level of effort depends on the proportional



gap between the wage and a reference wage. Following Summers (1988), the fair level of effort at firm  $i$  is given by

$$e^*(w_{it}, \bar{w}_{it}) = \left( \frac{w_{it} - \bar{w}_{it}}{\bar{w}_{it}} \right)^\sigma \quad (1.3)$$

for  $w_{it} > \bar{w}_{it}$  and  $0 < \sigma < 1$ , where  $w_{it}$  is the real product wage of firm  $i$  and  $\bar{w}$  is the real reference wage. The specification of the real reference wage will be discussed later. We define the elasticity of effort with respect to the real wage at the firm as  $\Omega_{it} = \frac{w_{it} e_{w_{it}}(w_{it}, \bar{w}_{it})}{e_{it}}$ . Given equation (1.3),  $\Omega_{it}$  can be written as

$$\Omega_{it} = \frac{\sigma w_{it}}{w_{it} - \bar{w}_{it}} \quad (1.4)$$

## 2.2 Firms

There is a continuum of identical firms on the unit interval. Each firm can hire up to one worker who produces an amount  $A_t e^{\varepsilon_{A,t}}$  where  $A_t$  is total factor productivity (we assume  $A = 1$  in equilibrium). The value of a filled job is

$$J_t = A_t e_t - w_t + \beta E_t[(1 - \tau_t)J_{t+1} + \tau_t V_{t+1}] \quad (2.1)$$

where  $V$  is the value of a vacancy. Firms must pay a real per-period cost of  $\gamma'$  at the start of each period to post a vacancy. Vacancies are then filled with probability  $q_t$ ; if the vacancy is filled, the new job match becomes productive in the following period. The value of an open vacancy is then

$$V_t = -\gamma' + \beta E_t[q_{t+1}J_{t+1} + (1 - q_{t+1})V_{t+1}] \quad (2.2)$$

## 2.3 The Labour Market

Employment evolves according to

$$N_{it} = (1 - \tau_t)N_{it-1} + h_{it} \quad (3.1)$$

where  $h_{i,t}$  is the number of workers hired and  $\tau_t$  is the exogenous job separation rate. The labour market is characterised by search frictions and so firms must post vacancies in order to hire workers. Aggregate hiring is determined by the matching function  $h_t = M(u_t, v_t)$  where  $M'(\cdot) > 0$ ,  $M''(\cdot) \leq 0$ ,  $h$  are aggregate hires,  $u$  is the number of job seekers and  $v$  are aggregate vacancies. We assume the matching function has constant returns to scale, so  $h_t = v_t M(\frac{u_t}{v_t}, 1)$ , hence the aggregate vacancy filling rate  $q_t$  is given by  $q_t = \frac{h_t}{V_t} = \bar{M}(\frac{u_t}{v_t})$ , where  $\bar{M}(\frac{u}{v}) = M(\frac{u}{v}, 1)$ . We define the vacancy filling rate for firm  $i$  as  $q_{it} = \frac{h_{it}}{v_{it}}$ . We assume that the number of workers hired by firm  $i$  is proportional to the relative number of vacancies it posts, so  $h_{it} = \frac{v_{it}}{v_t} h_t$ . As a result,  $q_{it} = q_t$  and so the vacancy filling rate is exogenous at the level of firm<sup>3</sup>. We assume that the real unit cost of posting a vacancy is constant. To proceed, We assume that the matching function has the Cobb-Douglas form  $h_t = mu_t^\alpha v_t^{1-\alpha}$ . Defining  $\theta = \frac{v}{u}$  as labour market tightness, total hiring is

$$h_t = mu_t \theta_t^{1-\alpha} \quad (3.2)$$

The probability of a firm filling a vacancy is

$$q_t = \frac{h_t}{v_t} = m \theta_t^{-\alpha} \quad (3.3)$$

while the probability that an unemployed worker finds a job is

$$f_t = \frac{h_t}{u_t} = \theta_t q_t \quad (3.4)$$

## 2.4 Optimal Wages and Employment

Imposing the free-entry condition  $V_t = 0$  on (2.2), we obtain

$$J_{t+1} = \frac{\gamma'}{\beta q_{t+1}} \quad (4.1)$$

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<sup>3</sup>We follow the literature and assume that firms seek to hire in every period.

Substituting (4.1) into (2.1), we obtain the job creation condition

$$A_t e_t - w_t - \lambda_t = 0 \quad (4.2)$$

where  $\lambda_t = \gamma \{ \frac{A_t}{q_t} - \beta(1 - \tau) E_t \frac{A_{t+1}}{q_{t+1}} \}$  is the real cost of hiring a worker ( $\gamma = \gamma'/\beta$ ). Equation (4.2) determines labour demand by equating the marginal product of labour to its' marginal cost. The wage that maximises the value of a filled job in (2.1) is

$$A_t e_{w_t} = 1 \quad (4.3)$$

where  $e_{w_t}$  is the derivative of the effort function with respect to the wage.

Dividing (4.3) by (4.2), we obtain

**Proposition.1** The optimality conditions for employment and the wage imply

$$\Omega_{it} = \frac{w_{it}}{w_{it} + \lambda_{it}} \quad (4.4)$$

At the optimum, the elasticity of effort with respect to the wage equals the ratio of the wage to the present value of the marginal cost of a new hire. Combining Proposition 1) with equation (1.4), we obtain

**Proposition.2** The optimal wage for the firm is

$$w_{it} = \frac{1}{1 - \sigma} \bar{w}_{it} + \frac{\sigma}{1 - \sigma} \lambda_{it} \quad (4.5)$$

The wage has two distinct components. The first is a pure efficiency wage effect in which the wage is a mark-up over the reference wage, where the mark-up reflects the strength of efficiency wages. The second reflects labour market frictions as the worker receives a proportion of hiring costs, where this proportion also reflects the strength of efficiency wage effects. This wage equation reflects an interaction between search frictions and efficiency wage effects. For example, an increase in the cost of posting a vacancy ( $\gamma$ ) increases  $\lambda_{it}$  and implies reduced employment and increased wages, leading

to increased effort. Therefore search frictions affect the optimal composition of effective labour.

The wage equation in (4.5) has interesting parallels with wage equations derived in models with search frictions e.g. Pissarides (2000). In both, the wage is an increasing function of search costs, but for different reasons. In a search frictions model, higher search costs increase the surplus from a job match. Since the wage bargain divides this surplus between workers and the firm, this is reflected in a higher wage<sup>4</sup>. In a model with search frictions and efficiency wages, by contrast, higher search costs induce firms to adjust the composition of effective labour by increasing effort and reducing employment, something that requires an increase in the wage. In (4.5), the wage also depends on efficiency wages; this effect is not present in the search frictions literature.

Propositions 1) and 2) extend results in the existing efficiency wage literature. The expression in (4.4) simplifies to the original Solow (1979) condition if there are no search costs. The first term in the wage equation in (4.5) is similar to expressions in Summers (1988) and Romer (2011). The second term in (4.5) extends these by adding a component that reflects labour market search<sup>5</sup>. Following Solow (1979), we can interpret these results as the outcome of minimization of the cost of effective labour. In this model, the cost of effective labour is  $(w_{it} + \lambda_{it})/e_{it}$ ; minimising this implies

$$\frac{e_{w_{it}}(w_{it} + \lambda_{it})}{e_{it}} = 1 \quad (4.6)$$

or

$$\Omega_{it}(1 + \frac{\lambda_{it}}{w_{it}}) = 1 \quad (4.7)$$

from which we can derive the modified Solow Condition in Proposition 1).

## 2.5 Specification of the Reference Wage

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<sup>4</sup>In a search frictions model, the surplus also reflects the value of employment to a worker. This aspect is irrelevant in this paper as the firm chooses the wage unilaterally.

<sup>5</sup>if there are no search costs, (4.5) simplifies to equation (10.15) in Romer (2011)

We consider two modelling approaches to the reference wage. One follows the literature in efficiency wages; the other one is based on search theory.

### 2.5.1 Social Norms and Fair Wages

The efficiency wage literature contains alternative specifications of the reference wage. Some refer to social norms; this is discussed by Katz (1986) who comments that "Akerlof (1982, 1984) and Solow (1979) argue that wage rigidity in the face of unemployment may be due to the importance of social wage norms". Other refer to a "gift exchange", eg Akerlof (1984), in which workers expect a "fair wage" from their employers. To capture these ideas, we express the reference wage as

$$\bar{w}_t^S = \Phi A_t^\rho \quad (4.8)$$

The social norm approach might suggest that the reservation wage is independent of the business cycle, in which case  $\rho = 0$ ; the "fair wage" approach might imply  $\rho > 0$  so workers expect a positive technology shock to be reflected in higher wages<sup>6</sup>. The parameter  $\Phi$  is a positive constant and  $\Phi \leq 1$ ; it measures the reference compensation level which workers feel well treated by their employer.

Substituting the reference wage (4.8) into the wage equation (4.5), we get

$$w_{i,t} = \frac{1}{1-\sigma} \Phi A_t^\rho + \frac{\sigma}{1-\sigma} \lambda_{i,t} \quad (4.9)$$

### 2.5.2 Reservation Wage

As stated above, when workers face a job offer, they either accept the job offer and receive the wage set by the firm or reject the job offer and stay

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<sup>6</sup>The reference wage (4.8) is similar to the wage determination in Blanchard and Gali (2010) and Michaillat (2012).

in the unemployment pool. To ensure that workers accept the wage offer, the flow value of being employed should be larger than the flow value of being unemployed. Following this idea, We define the reference wage as the annuity value of being unemployed, given by

$$\bar{w}_t^R = \frac{r}{1+r} U_t \quad (4.10)$$

Substituting the reference wage (4.10) into the wage equation (4.5), we get

$$w_{i,t} = \frac{1}{1-\sigma} \frac{r}{1+r} U_t + \frac{\sigma}{1-\sigma} \lambda_{i,t} \quad (4.11)$$

### 2.5.3 Steady-State Analysis

In steady-state,  $\bar{w}^R$  is the reservation wage, the wage at which  $L = U$ . Substitute the reservation wage into (1.1),

$$L - U = \frac{1+r}{r+\tau} (w - \bar{w}^R) \quad (4.12)$$

hence

$$\frac{w - \bar{w}^R}{\bar{w}^R} = \left( \frac{r+\tau}{r} \right) \left( \frac{L-U}{U} \right) \quad (4.13)$$

In steady-state,  $\frac{w - \bar{w}^R}{\bar{w}^R}$  is proportional to  $\frac{L-U}{U}$ , so effort reflects the value of employment relative to unemployment. Combining (4.10) and (4.12) with (1.2), we obtain

$$\bar{w}^R = \theta b + (1-\theta)w \quad (4.21)$$

where  $\theta = \frac{r+\tau}{r+\tau+f}$ . Hence the steady-state wage with this reference wage is

$$w = b + \frac{\sigma}{\theta - \sigma} (b + \lambda) \quad (4.22)$$

## 2.6 Summary of Key Equations

We summarize the model as follows. Two endogenous variables, the wage and labour market tightness, are determined by the wage equation (4.9) (or (4.11)) and the job creation condition (4.2). We pin down employment using the dynamic equation for employment (3.1). This makes our model comparable to the standard search and matching model. The only difference between the two is the production function and the wage specification. The model in this paper assumes output depends on workers' effort and firms set efficiency wages to minimize the cost of effective labour. Output in the standard search and matching model is exogenous and the wage is determined by Nash bargaining between the firm and the worker. Later in this chapter we use simulations to assess whether, using wage equations implied by efficiency wages, the search and matching model can replicate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of a plausible magnitude.

The key equations of the model include policy functions for job creation and the optimal wage, the specification for the reference wage and the equations for key labour market variables  $\{e_t, n_t, u_t, h_t, q_t, f_t, \theta_t\}$ . These equations are listed below:

Job creation condition,

$$A_t e_t - w_t - \lambda_t = 0 \quad (5.1)$$

The optimal wage set by the firm,

$$w_t = \frac{1}{1-\sigma} \bar{w}_t + \frac{\sigma}{1-\sigma} \lambda_t \quad (5.2)$$

The cost of hiring a worker,

$$\lambda_t = \gamma \left\{ \frac{1}{q_t} - \beta(1-\tau_t) E_t \frac{1}{q_{t+1}} \right\} \quad (5.3)$$

The effort function,

$$e_t = \left( \frac{w_t - \bar{w}_t}{\bar{w}_t} \right)^\sigma \quad (5.4)$$

The dynamic equation for employment,

$$n_t = (1 - \tau_{t-1})n_{t-1} + h_t \quad (5.5)$$

The definition of unemployment,

$$u_t = 1 - n_t \quad (5.6)$$

The matching function,

$$h_t = m u_t^\alpha v_t^{1-\alpha} \quad (5.7)$$

Labour market tightness,

$$\theta_t = \frac{v_t}{u_t} \quad (5.8)$$

The vacancy-filling rate,

$$q_t = m \theta_t^{-\alpha} \quad (5.9)$$

The job-finding rate,

$$f_t = \theta_t q_t \quad (5.10)$$

The reference wage based on social norms and fair wages,

$$\bar{w}_t^S = \Phi A_t^\rho \quad (5.11)$$

The reference wage based on search theory,

$$\bar{w}_t^R = \frac{r}{1+r} U_t \quad (5.12)$$

The value function for being employed,

$$L_t = w_t + \beta E_t[(1 - \tau_{t+1})L_{t+1} + \tau_{t+1}U_{t+1}] \quad (5.13)$$

The value function for being unemployed,

$$U_t = b + \beta E_t[f_t L_{t+1} + (1 - f_t)U_{t+1}] \quad (5.14)$$

### 2.6.1 Linearised Model



We obtain a linear approximation around the steady-state using a first-order Taylor series expansion. Details are contained in the appendix. Let  $\widehat{x}_t$  denote the percentage deviation of a variable  $X_t$  around its steady state and let  $\varepsilon_t^X$  denote the shock to variable  $X_t$ . Then the linearized form of the key equations are:

$$\widehat{e}_t + \varepsilon_t^A = \omega_1 \widehat{w}_t + (1 - \omega_1) \widehat{\lambda}_t \quad (5.15)$$

$$\widehat{w}_t = h_1 \widehat{\overline{w}_t} + (1 - h_1) \widehat{\lambda}_t \quad (5.16)$$

$$\widehat{\lambda}_t = -\lambda_1 \widehat{q}_t + (1 - \lambda_1) \widehat{q}_{t+1} + \lambda_2 \varepsilon_t^\tau \quad (5.17)$$

$$\widehat{e}_t = \sigma \frac{\overline{w}}{\overline{w} - \widehat{w}} (\widehat{w}_t - \widehat{\overline{w}_t}) \quad (5.18)$$

$$\widehat{n}_t = (1 - \tau) \widehat{n}_{t-1} + \tau \widehat{h}_t - \tau \varepsilon_{t-1}^\tau \quad (5.19)$$

$$\widehat{u}_t = \frac{-\overline{n}}{1 - \overline{n}} \widehat{n}_t \quad (5.20)$$

$$\widehat{h}_t = \alpha \widehat{u}_t + (1 - \alpha) \widehat{v}_t \quad (5.21)$$

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t \quad (5.22)$$

$$\widehat{q}_t = -\alpha \widehat{\theta}_t \quad (5.23)$$

$$\widehat{f}_t = \widehat{\theta}_t + \widehat{q}_t \quad (5.24)$$

$$\widehat{\overline{w}_t^S} = \rho \varepsilon_t^A \quad (5.25)$$

$$\widehat{\overline{w}_t^R} = \widehat{U}_t \quad (5.26)$$

$$\widehat{L}_t = a_1 \widehat{w}_t + a_2 \widehat{L}_{t+1} + (1 - a_1 - a_2) \widehat{U}_{t+1} + a_3 \varepsilon_{t+1}^\tau \quad (5.27)$$

$$\widehat{U}_t = b_1 \widehat{L}_{t+1} + b_2 \widehat{U}_{t+1} + b_3 \widehat{f}_t \quad (5.28)$$

where  $\omega_1 = \frac{\overline{w}}{\overline{w} + \overline{\lambda}}$ ,  $h_1 = \frac{\overline{w^r}}{\overline{w^r} + \sigma \overline{\lambda}}$ ,  $\lambda_1 = \frac{1}{1 - \beta(1 - \tau)}$ ,  $\lambda_2 = \frac{\beta \tau}{1 - \beta(1 - \tau)}$ ,  $a_1 = \frac{\overline{w}}{\overline{L}}$ ,  $a_2 = \beta(1 - \tau)$ ,  $a_3 = \frac{\beta \tau (\overline{U} - \overline{L})}{\overline{L}}$ ,  $b_1 = \frac{\beta \overline{f} \overline{L}}{\overline{U}}$ ,  $b_2 = \beta(1 - \overline{f})$ , and  $b_3 = \frac{\beta \overline{f} (\overline{L} - \overline{U})}{\overline{U}}$ .

## 3 Model Evaluation

### 3.1 Calibration

We calibrate the two models using established parameter values from the literature. These are summarised in Table 1. We normalize a time period to be one quarter, and set the risk-free interest rate to  $r = 0.012$ , equivalent to an annual risk-free interest rate of 0.048 (Shimer, 2005). Therefore the discount factor  $\beta$  is set to equal to 0.988. Following Shimer (2005) and Hall (2005), the job separation rate is set as  $\tau = 0.1$ , so on average 3.3 percent of employed workers exit employment every month. There is no consensus in literature on the calibrated values of the cost of posting a vacancy  $\gamma$  and the utility of leisure  $b$ . In the third chapter of this thesis, we argue that larger values of vacancy costs and the utility of leisure can both generate larger unemployment fluctuations. Since our goal is to evaluate the role of efficiency wage in solving unemployment volatility puzzle, we adopt the lower value of the vacancy cost and the utility of leisure in the literature. Following Shimer (2005), we assume the vacancy cost  $\gamma$  is 0.213 and the utility of leisure  $b$  is 0.4. We set the elasticity of matching with respect to unemployment  $\alpha$  equal to 0.5, a standard value in search literature, see Pissarides and Petrongolo (2001).

Table 1— Values of Calibrated Parameters

Parameter	Interpretation	Social Norm and Fair Wage	Reservation Wage
$\tau$	Exogenous Separation Rate	0.1	0.1
$\beta$	Discount Factor	0.988	0.988
$r$	Risk-Free Interest Rate	0.012	0.012
$b$	Utility of Leisure	0.4	0.4
$\gamma$	Vacancy Cost	0.213	0.213
$\sigma$	Elasticity of Effort	0.030	0.031
$m$	Matching Coefficient	1.31	0.9
$\alpha$	Matching Elasticity	0.5	0.5
$\Phi$	Coefficient of the Reference Wage	0.8	—
$\rho$	Wage Elasticity	0.5	—

There are no established values of  $\sigma$ ,  $m$ ,  $\rho$  and  $\Phi$  in the literature. We therefore set these so that both models match the average unemployment rate in the US, 5.8% (from BLS data:1948Q1 to 2014Q4), and average job finding rate of 0.55 per month (see Hagedorn and Mankoskii 2008). Doing so we obtained  $\sigma = 0.031$  and  $m = 0.9$  in the reservation wage model. For the model with social norm and fair wage, we set  $\sigma = 0.03$ ,  $m = 1.31$ , and  $\Phi = 0.8$ . The implied equilibrium unemployment rate and the job finding rate in both models are pretty close to the target, see Table 2. The value of wage elasticity with respect to productivity  $\rho$  is set to equal to 0.5, so the reference wage lies somewhere between the pure social norm and the pure fair wage extremes.

Table 2— Average Values of Endogenous Variables from Calibrations

Parameter	Interpretation	U.S Data	Social Norm and Fair Wage	Reservation Wage
$u$	Unemployment Rate	0.058	0.058	0.059
$\theta$	Labour Market Tightness		0.63	1.0
$f$	Job Finding Rate	0.55	0.54	0.53

We assume the shocks to labour productivity and job separation follow AR(1) process,

$$\varepsilon_t^A = \psi \varepsilon_{t-1}^A + \delta^A \quad (6.1)$$

$$\varepsilon_t^\tau = \varphi \varepsilon_{t-1}^\tau + \delta^\tau \quad (6.2)$$

We set  $\psi = 0.878$ , s.d ( $\delta^A$ ) = 0.01, and  $\varphi = 0.733$ , s.d ( $\delta^\tau$ ) = 0.05, so both the quarterly autocorrelation and standard deviation of labour productivity and job separation match U.S data, see Table 3.

Table 3— Statistics of Labour Productivity and Job Separation

Statistics	Labour Productivity	Job Separation
Quarterly Autocorrelation	0.878	0.733
Standard Deviation	0.020	0.075

Data Source: Shimer(2005), U.S monthly data, 1951-2003

## 3.2 Simulation Results

### 3.2.1 Volatility of Key Labour Market Variables

Table 4 describes the standard deviations of key labour market variables from U.S data and simulation results of three types of search and matching models. The results in the second and the third column are from Shimer (2005). The data source for the second column is U.S monthly data from 1951 to 2003, for details see pp.27-34 of Shimer (2005).

Table 4— Labour Productivity and Separation Shocks

	Standard Deviation of Key Labour Market Variables	U.S Data Shimer (2005)	Standard DMP Shimer (2005)	Social Norm and Fair Wage	Reservation Wage
$y$	Output	0.020	0.020	0.020	0.020
$s$	Job Separation	0.075	0.020	0.075	0.075
$u$	Unemployment Rate	0.190	0.031	0.196	0.181
$v$	Vacancy Rate	0.202	0.011	0.202	0.211
$\theta$	Market Tightness	0.382	0.037	0.373	0.365
$f$	Job Finding rate	0.118	0.014	0.186	0.183
$w$	Wage	—	—	0.021	0.002

The standard DMP column reports Shimer (2005)’s findings that the standard DMP model produces too little volatility of unemployment from realistic fluctuations in productivity and job separation. Comparing Shimer’s reports in the third column in Table 4 and the second column in Table 5 in which only labour productivity shock is considered, we find that the separation shock has only a trivial effect on the volatility of labour market tightness.

The last two columns of Table 4 show that the two types of search and matching model with efficiency wages can match the observed amount of volatility of unemployment, vacancy and labour market tightness well. In both models, the standard deviation of labour market tightness is almost 20 times as large as the standard deviation of average labour productivity. The standard deviation of unemployment and job vacancies are about 10 times as large as the standard deviation of average labour productivity. All those results are consistent with U.S data. The job finding rate in both models is more volatile than what the data indicates. This is probably because under the assumption of exogenous job separation, the separation shock generates a positive correlation between unemployment and vacancies, leading to labour market tightness being almost unchanged. Therefore the volatility of labour market tightness almost entirely relies on the fluctuations of job creation. In both models, the wage is much less volatile than the unemployment rate.

One might be concerned that the joint analysis of labour productivity and separation shocks may not say too much about the role of productivity shocks in unemployment and vacancies volatility. Next we carry out the simulation with only productivity shock. Results are reported in Table 5. In the absense of the shock to job separations, the decrease in the standard deviation of unemployment, job vacancies and labour market tightness in both models is trivial. This implies that the productivity shock is the major driving force behind labour market volatility.

Table 5— Labour Productivity Shocks

	Standard Deviation of Key Labour Market Variables	Standard DMP Shimer (2005)	Social Norm and Fair Wage	Reservation Wage
$y$	Output	0.020	0.020	0.020
$u$	Unemployment Rate	0.009	0.174	0.163
$v$	Vacancy Rate	0.027	0.196	0.204
$\theta$	Market Tightness	0.035	0.370	0.363
$f$	Job Finding rate	0.01	0.185	0.181
$w$	Wage	—	0.021	0.002

### 3.2.2 Variance Decomposition

Table 6 shows the variance decomposition of key labour market variables when the labour productivity shock and job separation shock are both taken into account. In both models, the separation shock plays a non-trivial role in terms of generating unemployment volatility. However for each labour market variable the productivity shock is unarguably the major driving force of their volatility.

Table 6— Variance Decomposition

Key Labour		Productivity Shock		Separation Shock	
Market Variables		Social Norm	Reservation	Social Norm	Reservation
		and Fair Wage	Wage	and Fair Wage	Wage
$u$	Unemployment Rate	79.31	80.82	20.61	19.18
$v$	Vacancy Rate	94.52	93.55	5.48	6.45
$\theta$	Market Tightness	98.75	98.73	1.25	1.27
$f$	Job Finding rate	98.75	98.73	1.25	1.27
$w$	Wage	100.00	99.42	0.00	0.58

### 3.2.3 Autocorrelation and Cross Correlations

The two models in the paper also outperform the standard DMP model in terms of replicating the autocorrelation of job vacancies and the correlations between unemployment, job vacancies, labour market tightness and the job finding rate.

The first column of the Table 7 shows the quarterly autocorrelations of key labour market variables observed in the data. The second column reports the same statistics obtained by simulating the standard DMP model. The quarterly autocorrelation of job vacancies in DMP model is less than one third of its counterpart in the data. By contrast, the same autocorrelations in our two models are much closer to the data. This might suggest that our models can better capture the dynamics of job creation.

Table 7— Quarterly Autocorrelation of Key Labour Market Variables

Key Labour		U.S Data	Standard DMP	Social Norm	Reservation
Market Variables		Shimer (2005)	Shimer (2005)	and Fair Wage	Wage
$u$	Unemployment Rate	0.936	0.933	0.848	0.930
$v$	Vacancy Rate	0.940	0.291	0.870	0.800
$\theta$	Market Tightness	0.941	0.878	0.876	0.876
$f$	Job Finding rate	0.908	0.878	0.876	0.876
$y$	Output	0.878	0.878	0.878	0.878
$s$	Job Separation	0.733	0.878	0.733	0.733

Table 8 shows the correlation matrix of key labour market variables in each model. U.S data indicates the correlation between unemployment and job vacancies is -0.894, so the two variables are strongly negatively correlated. However the standard DMP model cannot capture this feature, the correlation between two variables in the DMP model is only -0.427. Our two models do a better job on this. The correlations between unemployment and job vacancies in the social norms and fair wages model is -0.76 and in the reservation wage model is -0.73. The standard DMP model also cannot capture the observed correlation between job vacancies and labour market tightness and the observed correlation between job vacancies and job finding rate. Here again, our two models outperform the standard DMP model. This reflects the fact that our search models with efficiency wages better capture labour market dynamics.

Table 8— Matrix of Correlation of Key Labour Market Variables

		$u$	$v$	$\theta$	$f$
U.S Data	$u$	1	-0.894	-0.971	-0.949
Standard DMP	$u$	1	-0.427	-0.964	-0.964
Fair Wage	$u$	1	-0.760	-0.936	-0.936
Reservation Wage	$u$	1	-0.730	-0.919	-0.919
U.S Data	$v$	—	1	0.975	0.897
Standard DMP	$v$	—	1	0.650	0.650
Fair Wage	$v$	—	1	0.940	0.940
Reservation Wage	$v$	—	1	0.941	0.941
U.S Data	$\theta$	—	—	1	0.948
Standard DMP	$\theta$	—	—	1	1
Fair Wage	$\theta$	—	—	1	1
Reservation Wage	$\theta$	—	—	1	1
U.S Data	$f$	—	—	—	1
Standard DMP	$f$	—	—	—	1
Fair Wage	$f$	—	—	—	1
Reservation Wage	$f$	—	—	—	1



## 4 Conclusion

In this chapter we have added a Solow-type efficiency wage effect to an otherwise standard search frictions model. We have derived a simple generalisation of the Solow Condition, which we used to express the wage as the sum of components reflecting search frictions and efficiency wages. We found that this alternative type of search frictions model delivers the same unemployment and job vacancies fluctuations as in U.S data. We also found that the models capture the labour market dynamics well.

We would argue that our results are interesting but not definitive. We would wish to develop our work, in two main directions. First, we have replaced wage bargaining with efficiency wages; the logical next step is to combine both in the same model and analyse interactions between them. Second, a natural extension of our work would analyse the effect of the aggregate demand on unemployment and job vacancies fluctuations in a general equilibrium set-up. We hope to address these issues in future work.

## APPENDIX OF CHAPTER 1

### The Linear Approximation

#### *The Job Creation Condition*

$$A_t e_t = w_t + \lambda_t$$

First, take the natural log on both sides of job creation condition,

$$\ln A_t + \ln e_t = \ln(w_t + \lambda_t)$$

then rewrite the job creation condition by replacing each variable with its first-order taylor series expansion around the steady state:

$$\varepsilon_t^A + \ln e + \frac{1}{e}(e_t - e) = \ln(w + \lambda) + \frac{1}{w + \lambda}(w_t - w) + \frac{1}{w + \lambda}(\lambda_t - \lambda)$$

since  $\ln e = \ln(w + \lambda)$ , it follows that

$$\varepsilon_t^A + \frac{1}{e}(e_t - e) = \frac{1}{w + \lambda}(w_t - w) + \frac{1}{w + \lambda}(\lambda_t - \lambda)$$

let  $\hat{x}_t$  denote the percentage deviation of a variable  $X_t$  around its steady state, rewrite the above equation as

$$\varepsilon_t^A + \hat{e}_t = \frac{w}{w + \lambda} \hat{w}_t + \frac{\lambda}{w + \lambda} \hat{\lambda}_t$$

#### *The optimal wage*

$$w_t = \frac{1}{1 - \sigma} w_t^r + \frac{\sigma}{1 - \sigma} \lambda_t$$

One can write this as

$$\ln w_t = \ln\left(\frac{1}{1 - \sigma} w_t^r + \frac{\sigma}{1 - \sigma} \lambda_t\right)$$

then rewrite each variable as its first-order taylor series expansion around

the steady state

$$\ln w + \frac{1}{w}(w_t - w) = \ln\left(\frac{1}{1-\sigma}w^r + \frac{\sigma}{1-\sigma}\lambda\right) + \frac{\frac{1}{1-\sigma}}{\frac{1}{1-\sigma}w^r + \frac{\sigma}{1-\sigma}\lambda}(w_t^r - w^r) + \frac{\frac{\sigma}{1-\sigma}}{\frac{1}{1-\sigma}w^r + \frac{\sigma}{1-\sigma}\lambda}(\lambda_t - \lambda)$$

since  $\ln w = \ln(\frac{1}{1-\sigma}w^r + \frac{\sigma}{1-\sigma}\lambda)$ , it follows that

$$\frac{1}{w}(w_t - w) = \frac{\frac{1}{1-\sigma}}{\frac{1}{1-\sigma}w^r + \frac{\sigma}{1-\sigma}\lambda}(w_t^r - w^r) + \frac{\frac{\sigma}{1-\sigma}}{\frac{1}{1-\sigma}w^r + \frac{\sigma}{1-\sigma}\lambda}(\lambda_t - \lambda)$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\hat{w}_t = \frac{w^r}{w^r + \sigma\lambda}\hat{w}_t^r + \frac{\sigma\lambda}{w^r + \sigma\lambda}\hat{\lambda}_t$$

*The hiring cost*

$$\lambda_t = \gamma\left[\frac{1}{q_t} - \beta(1 - \tau_t)E_t\frac{1}{q_{t+1}}\right]$$

Take the natural log on both sides of equation

$$\ln \lambda_t = \ln \gamma + \ln\left[\frac{1}{q_t} - \beta(1 - \tau_t)E_t\frac{1}{q_{t+1}}\right]$$

then rewrite each variable as its first-order taylor series expansion around the steady state

$$\begin{aligned} \ln \lambda + \frac{1}{\lambda}(\lambda_t - \lambda) &= \ln \gamma + \ln\left[\frac{1}{q} - \beta(1 - \tau)\frac{1}{q}\right] + \frac{q}{1 - \beta(1 - \tau)}\frac{-1}{q^2}(q_t - q) \\ &+ \frac{q}{1 - \beta(1 - \tau)}\frac{\beta(1 - \tau)}{q^2}(q_{t+1} - q) + \frac{q}{1 - \beta(1 - \tau)}\frac{\beta}{q}(\tau_t - \tau) \end{aligned}$$

since  $\ln \lambda = \ln \gamma + \ln[\frac{1}{q} - \beta(1 - \tau)\frac{1}{q}]$ , it follows that

$$\frac{1}{\lambda}(\lambda_t - \lambda) = \frac{q}{1 - \beta(1 - \tau)} \left[ \frac{-1}{q^2}(q_t - q) + \frac{\beta(1 - \tau)}{q^2}(q_{t+1} - q) + \frac{\beta}{q}(\tau_t - \tau) \right]$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\hat{\lambda}_t = \frac{-1}{1 - \beta(1 - \tau)} \hat{q}_t + \frac{\beta(1 - \tau)}{1 - \beta(1 - \tau)} \hat{q}_{t+1} + \frac{\beta\tau}{1 - \beta(1 - \tau)} \varepsilon_t^\tau$$

*The effort function*

$$e_t = \left( \frac{w_t - \bar{w}_t}{\bar{w}_t} \right)^\sigma$$

Take the natural log on both sides of equation

$$\ln e_t = \sigma \ln \left( \frac{w_t - \bar{w}_t}{\bar{w}_t} \right)$$

then rewrite each variable as its first-order taylor series expansion around the steady state

$$\ln e + \frac{1}{e}(e_t - e) = \sigma \ln \left( \frac{w - \bar{w}}{\bar{w}} \right) + \sigma \frac{\bar{w}}{w - \bar{w}} \frac{1}{\bar{w}}(w_t - w) + \sigma \frac{\bar{w}}{w - \bar{w}} \frac{-w}{\bar{w}^2}(\bar{w}_t - \bar{w})$$

since  $\ln e = \sigma \ln \left( \frac{w - \bar{w}}{\bar{w}} \right)$ , it follows that

$$\frac{1}{e}(e_t - e) = \sigma \frac{\bar{w}}{w - \bar{w}} \frac{1}{\bar{w}}(w_t - w) + \sigma \frac{\bar{w}}{w - \bar{w}} \frac{-w}{\bar{w}^2}(\bar{w}_t - \bar{w})$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\hat{e}_t = \sigma \frac{\bar{w}}{\bar{w} - w^r} (\hat{w}_t - \hat{w}_t^r)$$

*The dynamic equation for employment*

$$n_t = (1 - \tau_{t-1})n_{t-1} + h_t$$

One can write this as

$$\ln n_t = \ln[(1 - \tau_{t-1})n_{t-1} + h_t]$$

then rewrite each variable as its first-order taylor series expansion around the steady state

$$\ln n + \frac{1}{n}(n_t - n) = \ln[(1 - \tau)n + h] + \frac{1 - \tau}{n}(n_{t-1} - n) + \frac{-n}{n}(\tau_{t-1} - \tau) + \frac{1}{n}(h_t - h)$$

since  $\ln n = \ln[(1 - \tau)n + h]$ , we have

$$\frac{1}{n}(n_t - n) = \frac{1 - \tau}{n}(n_{t-1} - n) + \frac{-n}{n}(\tau_{t-1} - \tau) + \frac{1}{n}(h_t - h)$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\hat{n}_t = (1 - \tau)\hat{n}_{t-1} + \tau\hat{h}_t - \tau\varepsilon_{t-1}^\tau$$

*The definition of unemployment*

$$u_t = 1 - n_t$$

Take the natural log on both sides of equation

$$\ln u_t = \ln(1 - n_t)$$

then rewrite each variable as its first-order taylor series expansion around the steady state

$$\ln u + \frac{1}{u}(u_t - u) = \ln(1 - n) + \frac{-1}{1 - n}(n_t - n)$$

since  $\ln u = \ln(1 - n)$ , it follows that

$$\frac{1}{u}(u_t - u) = \frac{-1}{1 - n}(n_t - n)$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\hat{u}_t = \frac{-n}{1 - n}\hat{n}_t$$

*The matching function*

$$h_t = m u_t^\alpha v_t^{1-\alpha}$$

Take the natural log on both sides of equation

$$\ln h_t = \ln m + \alpha \ln u_t + (1 - \alpha) \ln v_t$$

then replace each variable with its first-order taylor series expansion around the steady state

$$\ln h + \frac{1}{h}(h_t - h) = \ln m + \alpha \ln u + \alpha \frac{1}{u}(u_t - u) + (1 - \alpha) \ln v + (1 - \alpha) \frac{1}{v}(v_t - v)$$

since  $\ln h = \ln m + \alpha \ln u + (1 - \alpha) \ln v$ , we have

$$\frac{1}{h}(h_t - h) = \alpha \frac{1}{u}(u_t - u) + (1 - \alpha) \frac{1}{v}(v_t - v)$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\hat{h}_t = \alpha \hat{u}_t + (1 - \alpha) \hat{v}_t$$

*The labour market tightness*

$$\theta_t = \frac{v_t}{u_t}$$

Take the natural log on both sides of equation and replace each variable

with its first-order taylor series expansion around the steady state

$$\ln \theta + \frac{1}{\theta}(\theta_t - \theta) = \ln v + \frac{1}{v}(v_t - v) - \ln u - \frac{1}{u}(u_t - u)$$

Rearrange the equation and use  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state,

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t$$

*The vacancy-filling rate*

$$q_t = m\theta_t^{-\alpha}$$

Take the natural log on both sides of equation and replace each variable with its first-order taylor series expansion around the steady state

$$\ln q + \frac{1}{q}(q_t - q) = \ln m - \alpha \ln \theta - \alpha \frac{1}{\theta}(\theta_t - \theta)$$

Rearrange the equation and use  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state,

$$\hat{q}_t = -\alpha \hat{\theta}_t$$

*The job-finding rate*

$$f_t = \theta_t q_t$$

Take the natural log on both sides of equation and replace each variable with its first-order taylor series expansion around the steady state

$$\ln f + \frac{1}{f}(f_t - f) = \ln \theta + \frac{1}{\theta}(\theta_t - \theta) + \ln q + \frac{1}{q}(q_t - q)$$

Rearrange the equation and use  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state,

$$\hat{f}_t = \hat{\theta}_t + \hat{q}_t$$

*The reference wage based on social norm and fair wage*

$$\bar{w}_t = \Phi A_t^\rho$$

Take the natural log on both sides of equation and replace each variable with its first-order taylor series expansion around the steady state

$$\ln \bar{w} + \frac{1}{\bar{w}}(\bar{w}_t - \bar{w}) = \ln \Phi + \rho \varepsilon_t^A$$

Rearrange the equation and use  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state,

$$\widehat{\bar{w}_t} = \rho \varepsilon_t^A$$

*The reference wage based on reservation wage*

$$\bar{w}_t = \frac{r}{1+r} U_t$$

Take the natural log on both sides of equation and replace each variable with its first-order taylor series expansion around the steady state

$$\ln \bar{w} + \frac{1}{\bar{w}}(\bar{w}_t - \bar{w}) = \ln \frac{r}{1+r} + \ln U + \frac{1}{U}(U_t - U)$$

Rearrange the equation and use  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state,

$$\widehat{\bar{w}_t} = \hat{U}_t$$

*The value function for being employed*



$$L_t = w_t + \beta E_t[(1 - \tau_{t+1})L_{t+1} + \tau_{t+1}U_{t+1}]$$

Take the natural log on both sides of equation

$$\ln L_t = \ln\{w_t + \beta E_t[(1 - \tau_{t+1})L_{t+1} + \tau_{t+1}U_{t+1}]\}$$

then rewrite each variable as its first-order taylor series expansion around the steady state

$$\begin{aligned} \ln L + \frac{1}{L}(L_t - L) &= \ln\{w + \beta[(1 - \tau)L + \tau U]\} + \frac{\beta(U - L)}{L}(\tau_t - \tau) \\ &\quad + \frac{\beta(1 - \tau)}{L}(L_{t+1} - L) + \frac{\beta\tau}{L}(U_{t+1} - U) + \frac{1}{L}(w_t - w) \end{aligned}$$

since  $\ln L = \ln\{w + \beta[(1 - \tau)L + \tau U]\}$ , it follows that

$$\frac{1}{L}(L_t - L) = \frac{\beta(U - L)}{L}(\tau_t - \tau) + \frac{\beta(1 - \tau)}{L}(L_{t+1} - L) + \frac{\beta\tau}{L}(U_{t+1} - U) + \frac{1}{L}(w_t - w)$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\widehat{L}_t = \frac{w}{L}\widehat{w}_t + \beta(1 - \tau)\widehat{L}_{t+1} + \frac{\beta\tau U}{L}\widehat{U}_{t+1} + \frac{\beta\tau(U - L)}{L}\varepsilon_{t+1}^\tau$$

*The value function for being unemployed*

$$U_t = b + \beta E_t[f_t L_{t+1} + (1 - f_t)U_{t+1}]$$

Take the natural log on both sides of equation

$$\ln U_t = \ln\{b + \beta E_t[f_t L_{t+1} + (1 - f_t)U_{t+1}]\}$$

then rewrite each variable as its first-order taylor series expansion around

the steady state

$$\begin{aligned} \ln U + \frac{1}{U}(U_t - U) &= \ln\{b + \beta[fL + (1 - f)U]\} \\ &+ \frac{\beta(L - U)}{U}(f_t - f) + \frac{\beta f}{U}(L_{t+1} - L) + \frac{\beta(1 - f)}{U}(U_{t+1} - U) \end{aligned}$$

since  $\ln U = \ln\{b + \beta[fL + (1 - f)U]\}$ , we have

$$\frac{1}{U}(U_t - U) = \frac{\beta(L - U)}{U}(f_t - f) + \frac{\beta f}{U}(L_{t+1} - L) + \frac{\beta(1 - f)}{U}(U_{t+1} - U)$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\widehat{U}_t = \frac{\beta f L}{U} \widehat{L}_{t+1} + \beta(1 - f) \widehat{U}_{t+1} + \frac{\beta f(L - U)}{U} \widehat{f}_t$$

## Chapter 2

# Equilibrium Unemployment and Labour Market Volatility

### Abstract

This chapter analyses labour market volatility in a model that adds a Shapiro-Stiglitz type efficiency wage to an otherwise standard search and matching frictions model of the labour market. In this model, firms set an efficiency wage to deter workers from shirking. Our analysis indicates that the wage is less responsive to productivity shocks than with Nash bargained wages. We argue that our model outperforms the standard search frictions model in two key aspects: (i) our model can match the observed business-cycle-frequency fluctuations in unemployment and job vacancies; (ii) our model also better captures labour market dynamics.

# 1 Introduction

There are two broad types of efficiency wage model. Following Solow (1979) and Summers (1988), one type of model assumes that effort is a continuous variable. The implication of this type of efficiency wage model on the labour market volatility has been discussed in the previous chapter. The other, following Shapiro and Stiglitz (1984) assumes that effort is binary, corresponding to a worker choosing to shirk or not. This chapter explores the implications of this alternative type of efficiency wage on labour market volatility.

There is a small existing literature that does this. Uhlig and Xu (1996) incorporate the shirking mechanism into a real business cycle (RBC) model. They argue that the shirking mechanism can explain the observed volatility of labour input only if productivity shocks are implausibly large. Gomme (1999) explores wage behavior in a RBC-type model with a shirking mechanism which assumes that shirking workers are always detected and dismissed. This paper concludes that a shirking mechanism can generate dampened but still strongly procyclical wage behavior. Alexopoulos (2006) incorporates a shirking mechanism into a limited participation DSGE model in which firms pay workers part of their wage upfront and do not pay the balance to workers who are detected as shirkers. Using this wage deferment process, the model is able to capture the observed response of macro-variables including employment to monetary policy shocks. None of these models explores the unemployment volatility puzzle. Also none of them includes search and matching frictions.

In this paper we incorporate the shirking mechanism developed by Shapiro and Stiglitz (1984) into an otherwise standard search and matching model and use this to address the volatility puzzle. To the best of our knowledge, no other paper does this. As in the first chapter we consider the influential paper by Shimer (2005), who documents the inability of simulations of a parameterized version of the standard DMP model where wages are determined through wage bargaining to match the volatilities of unemployment, job vacancies, and labour market tightness in US data for 1951-2003. We

consider whether our alternative model, calibrated to match the average unemployment rate and job finding rate in the U.S, can better capture the observed labour market volatilities.

The paper has three contributions. First, we develop a simple analytical model by adding the shirking mechanism to an otherwise standard search frictions model. The wage in our model shares some determinants with the standard DMP model: the utility of leisure, the rate at which job matches are dissolved and the job finding rate. However we find that the wage elasticity with respect to labour productivity has rather different determinants in the two models. In the standard DMP model, the wage responds to productivity shocks for two reasons. One is that productivity affects the wage level directly since productivity affects the size of match surplus. The other one is the response of labour market tightness to a productivity shock. A positive productivity shock makes job creation more appealing to firms. So more job vacancies are posted. This pulls down the unemployment rate and increases labour market tightness. The increase in labour market tightness shortens unemployment duration, raising the value of being unemployed to the worker, and therefore raising worker's threat point in wage bargaining. In our model, by contrast, the wage responds to a productivity shock if and only if the job finding rate responds to productivity. If the job finding rate increases after a positive productivity shock, then the relative value of being unemployed to a worker also increases. This reduces the potential loss of being fired if the worker chooses to shirk since it becomes easier to find a new job. As a result, the firm has to raise the wage to increase the threat point faced by workers.

Empirical evidence shows that the job finding rate is less variable than labour market tightness under productivity shocks. The search frictions model captures this fact when a constant return to scale of matching process is assumed. Therefore, with reasonable calibrations, the wage in our model exhibits smaller variations than Nash bargained wage and this helps our model to capture the observed fluctuations of unemployment and job vacancies.

Second, using standard values for the structural parameters, we show

that our model can match the observed value of key labour market indicators in the U.S economy, such as the average unemployment rate and the rates at which unemployed workers find jobs. Moreover, our model can generate the observed business-cycle-frequency fluctuations in unemployment, job vacancies and the vacancy-unemployment ratio in response to shocks of a plausible magnitude. Third, we show that, again using standard values for structural parameters, our model outforms the standard DMP model in terms of capturing some dynamic features of the U.S labour market, such as the correlations and autocorrelation of key labour market indicators.

The remainder of the paper is structured as follows. We outline the model in section 2) explaining the objectives of workers and firms and explaining job matching and labour market flows. Then we characterise the model by deriving the optimality conditions for vacancy creations and wages. In section 3) we calibrate and simulate the model. Section 4) concludes.

## 2 The Model

### 2.1 Workers

There is a continuum of identical workers on the unit interval, all of whom dislike putting forth effort, but enjoy consuming goods. Each worker inelastically supplies one unit of labour in every period and consumes all the income they earn. We write an individual's utility function as  $w_t - e - \chi$ , where  $w_t$  is the wage received and  $\chi$  is the utility of leisure<sup>7</sup> and  $e$  is the disutility of effort. For simplicity, we assume that workers can provide either no effort (in this case  $e = 0$ )<sup>8</sup>, or some fixed positive level of effort which leads to  $e = 1$ .

In period  $t$  a worker is in one of three states. If they are employed and not shirking, they earn a wage  $w_t$  but incur disutility of effort of  $e$ . If they are employed and shirking, they earn the same wage but do not suffer the

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<sup>7</sup>Workers lose the utility of leisure when going to work, no matter they provide effort or not.

<sup>8</sup>Alternatively, we could assume that effort is positive but less than 1 if the worker shirks. However, this wouldn't affect the no-shirking constraint faced by the firm.

disutility of effort. If they are unemployed, they earn real unemployment benefits  $b$ . If unemployed, an individual finds a job with probability  $f_t$ . Therefore the expected length of the unemployment spell an individual must face is  $1/f_t$ .

At the end of each period, existing job matches exogenously separate with probability  $\tau_t$ . The separation rate is stochastic:  $\tau_t = \tau e^{\varepsilon_t^\tau}$  where  $\varepsilon_t^\tau$  is a separation rate shock and  $\varepsilon_t^\tau = \rho^\tau \varepsilon_{t-1}^\tau + \eta_t^\tau$ , where  $\eta_t^\tau$  is distributed as  $N(0, \sigma_\tau^2)$ . In addition, workers who shirk are detected and dismissed with probability  $\delta$ . Following Shapiro and Stiglitz (1984), we also assume that exogenous job separation and the firing of a shirking worker are two mutually exclusive events. So when a worker shirks, the probability he leaves the job is  $\tau + \delta$ .<sup>9</sup>

The workers choose their effort level to maximize their discounted utility stream. This involves comparison of the utility from shirking with the utility from not shirking. The value function for being employed and not shirking is

$$L_t = w_t - e - \chi + \beta E_t[(1 - \tau_t)M_{t+1} + \tau_t U_{t+1}] \quad (2.1)$$

where  $\beta$  is the discount factor. We assume that  $\beta = 1/(1 + r)$ , in which  $r$  is the risk-free real interest rate. We define  $M_t$  as  $\max(L_t, S_t)$ , in which  $S_t$  is the value function for being employed and shirking.  $S_t$  is given by

$$S_t = w_t - \chi + \beta E_t[(1 - \tau_t - \delta)M_{t+1} + (\tau_t + \delta)U_{t+1}] \quad (2.2)$$

The value of being unemployed is

$$U_t = b + \beta E_t[f_t M_{t+1} + (1 - f_t)U_{t+1}] \quad (2.3)$$

The worker will choose not to shirk if and only if  $L_t \geq S_t$ . This is the so-called no-shirking condition (NSC), which, using (2.1) and (2.2), can be

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<sup>9</sup> An alternative specification is to assume exogenous job separation and action of firing a shirking worker are two independent events. So when a worker shirks, the probability he leaves the job is  $\tau + \delta - \tau\delta$ . We find that the results are robust with respect to this alternative specification.

written as

$$\beta\delta E_t(M_{t+1} - U_{t+1}) \geq e \quad (2.4)$$

The intuition behind the NSC is that a worker will choose not to shirk if and only if the expected discounted loss due to shirking is no less than the disutility of effort. Equation (2.4) is a discrete time form of the no-shirking condition in Shapiro and Stiglitz (1984).

## 2.2 Firms

There is a continuum of identical firms on the unit interval. Each firm can hire up to one worker who produces an amount  $A_t$  if they do not shirk and nothing if they do shirk, where  $A_t = e^{\varepsilon_t^A}$  is total factor productivity;  $\varepsilon_t^A$  is a technology shock where  $\varepsilon_t^A = \psi\varepsilon_{t-1}^A + \eta_t^A$ , where  $\eta_t^A$  is distributed as  $N(0, \sigma_s^2)$ . The value of a filled job to the firm is

$$J_t = A_t - w_t + \beta E_t[(1 - \tau_t)J_{t+1} + \tau_t V_{t+1}] \quad (2.5)$$

where  $V$  is the value of a vacancy. Firms must pay a real cost of  $\gamma'$  to post a vacancy. Vacancies are then filled at the start of the next period with probability  $q$ ; if the vacancy is filled, the new job match becomes productive immediately. The value of an open vacancy is then

$$V_t = -\gamma' + \beta E_t[q_{t+1}J_{t+1} + (1 - q_{t+1})V_{t+1}] \quad (2.6)$$

The value functions (2.5) and (2.6) are different to Shapiro and Stiglitz (1984) in the following respects: (i) firms pay a real cost to post a vacancy; (ii) the rate of filling a job vacancy is endogenized. Next we turn to characterise the labour market.

## 2.3 The Labour Market

Employment evolves according to

$$N_{it} = (1 - \tau_t)N_{it-1} + h_{it} \quad (2.7)$$



where  $h_{i,t}$  is the number of workers hired and  $\tau_t$  is the exogenous job separation rate. We assume  $\tau_t = \tau \varepsilon_t^\tau$  where  $\varepsilon_t^\tau$  is a separation shock. The labour market is characterised by search frictions and so firms must post vacancies in order to hire workers. Aggregate hiring is determined by the matching function  $h_t = M(u_t, v_t)$  where  $M'(\cdot) > 0$ ,  $M''(\cdot) \leq 0$ ,  $h$  are aggregate hires,  $u$  is the number of job seekers and  $v$  are aggregate vacancies. We assume the matching function has constant returns to scale, so  $h_t = v_t M(\frac{u_t}{v_t}, 1)$ , hence the aggregate vacancy filling rate  $q_t$  is given by  $q_t = \frac{h_t}{v_t} = \bar{M}(\frac{u_t}{v_t})$ , where  $\bar{M}(\frac{u}{v}) = M(\frac{u}{v}, 1)$ . We define the vacancy filling rate for firm  $i$  as  $q_{it} = \frac{h_{it}}{v_{it}}$ . We assume that the number of workers hired by firm  $i$  is proportional to the relative number of vacancies it posts, so  $h_{it} = \frac{v_{it}}{v_t} h_t$ . As a result,  $q_{it} = q_t$  and so the vacancy filling rate is exogenous at the level of firm<sup>10</sup>. We assume that the real unit cost of posting a vacancy is constant. To proceed, We assume that the matching function has the Cobb-Douglas form  $h_t = m u_t^\alpha v_t^{1-\alpha}$ . Defining  $\theta = \frac{v}{u}$  as labour market tightness, total hiring is

$$h_t = m u_t \theta_t^{1-\alpha} \quad (2.8)$$

The probability of a firm filling a vacancy is

$$q_t = \frac{h_t}{v_t} = m \theta_t^{-\alpha} \quad (2.9)$$

while the probability that an unemployed worker finds a job is

$$f_t = \frac{h_t}{u_t} = \theta_t q_t \quad (2.10)$$

## 2.4 Market Equilibrium

We now turn to the determination of the equilibrium wage and employment levels. We assume that both unemployment benefits ( $b$ ) and the monitoring technology ( $\delta$ ) are exogenous to firms. Each individual firm chooses the optimal wage in each period. According to the no-shirking condition, the

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<sup>10</sup> We follow the literature and assume that firms seek to hire in every period.

firm chooses the lowest wage at which the worker decides not to shirk. This implies  $L = S$  in every period. Rewriting the NSC, we obtain

$$e = \beta\delta E_t(L_{t+1} - U_{t+1}) \quad (2.11)$$

Using (2.1) and (2.3) we obtain

$$E_t(L_t - U_t) = E_t[w_t - e - \chi - b + \beta(1 - \tau_t - f_t)(L_{t+1} - U_{t+1})] \quad (2.12)$$

Combining (2.11) and (2.12), the optimal wage is given by

$$w_t = b + \chi + e[1 + \frac{1 - \beta(1 - \tau_t - f_t)}{\beta\delta}] \quad (2.13)$$

Substituting  $\beta = \frac{1}{1+r}$  into (2.13), we obtain

$$w_t = b + \chi + e(1 + \frac{r + \tau_t + f_t}{\delta}) \quad (2.14)$$

This is equivalent to the no-shirking condition found in the more common continuous time version of the Shapiro-Stiglitz model. Equilibrium occurs when each firm, taking as given the job separation rate and job finding rate, finds it optimal to offer the going wage rather than a different wage. This wage determination is the same as in Shapiro and Stiglitz (1984).

Free entry of firms drives rents from vacant jobs to zero. Imposing  $V = 0$  on (2.6), we obtain

$$J_{t+1} = \frac{\gamma'}{\beta q(\theta)_{t+1}} \quad (2.15)$$

Condition (2.15) states that the expected profit from a new job is equal to the expected cost of hiring a worker. Substituting condition (2.15) into (2.5), we obtain

$$A_t - w_t - \lambda_t = 0 \quad (2.16)$$

where  $\lambda_t = \gamma\{\frac{1}{q_t} - \beta(1 - \tau)E_t\frac{1}{q_{t+1}}\}$  is the real cost of hiring a worker ( $\gamma = \gamma'/\beta$ ). Equation (2.16) determines labour demand by equating the marginal product of labour to its marginal cost. In equilibrium  $q_t = q_{t+1}$ . Substitute

this and  $\beta = \frac{1}{1+r}$  into (2.16), we obtain

$$A_t - w_t - \frac{(r + \tau_t)\gamma}{q(\theta)_t} = 0 \quad (2.17)$$

In each period, the number of workers who enter unemployment in each period is  $\tau_t(1 - u_t)$ , and the number who leave unemployment is  $f(\theta)_t u_t$ . In equilibrium unemployment is constant, so the two flows are equal,

$$\tau_t(1 - u_t) = f(\theta)_t u_t \quad (2.18)$$

substituting this into (2.14), the wage can be rewritten as

$$w_t = e + \chi + b + \frac{e}{\delta} \left( r + \frac{\tau_t}{u_t} \right) \quad (2.19)$$

## 2.5 Comparison with the Standard Search and Matching Model

In the standard search and matching model, workers always put forth effort. Therefore workers do not shirk and  $\delta = 0$ . Workers' utility is  $w_t$ , as there is no disutility of effort. The value function for being employed is

$$L_t = w_t + \beta E_t[(1 - \tau_t)L_{t+1} + \tau_t U_{t+1}] \quad (2.20)$$

Workers have the same value function as (2.3) when they are unemployed. Firms also have the same value functions as (2.5) and (2.6). So the job match always yields a surplus  $S_t$ , defined as

$$S_t = J_t + L_t - U_t \quad (2.21)$$

The match surplus is split between the firm and the worker according to Nash bargaining. The wage is chosen to maximize

$$\max(L_t - U_t)^\phi J_t^{1-\phi} \quad (2.22)$$

where  $\phi$  is the worker's relative bargaining power. The optimal condition is

$$L_t - U_t = \phi(J_t + L_t - U_t) \quad (2.23)$$

Substituting (2.3) and (2.20) into (2.23), we obtain

$$L_t - U_t = w_t - \chi - b + \beta(1 - \tau_t - f_t)E_t(L_{t+1} - U_{t+1}) \quad (2.24)$$

The resultant wage is given by

$$w_t = b + \chi + \frac{\phi}{1 - \phi} \gamma \left[ \frac{1}{q_t} - \beta(1 - \tau_t - f_t) \frac{1}{q_{t+1}} \right] \quad (2.25)$$

In equilibrium,  $q_t = q_{t+1}$ . (2.25) becomes

$$w_t = b + \chi + \frac{\phi}{1 - \phi} [1 - \beta(1 - \tau_t - f_t)] \frac{\gamma}{q_t} \quad (2.26)$$

Substitute  $\beta = \frac{1}{1+r}$  into (2.26),

$$w_t = b + \chi + \frac{\phi}{1 - \phi} (r + \tau_t) \frac{\gamma'}{q_t} + \frac{\phi}{1 - \phi} \gamma' \theta_t \quad (2.27)$$

In equilibrium, job creation condition satisfies

$$A_t - w_t - \frac{(r + \tau_t) \gamma'}{q(\theta)_t} = 0 \quad (2.28)$$

Substituting (2.28) into (2.27), the wage equation now can be written as

$$w_t = (1 - \phi)(b + \chi) + \phi(A_t + \gamma' \theta_t) \quad (2.29)$$

This is equivalent to the wage equation in the continuous time version of the search and matching model, see Pissarides (2000).

### 2.5.1 Wage Elasticity

The literature argues that a non-trivial response of the wage to labour pro-

ductivity is a potential reason why the standard DMP model cannot match the observed fluctuations in unemployment and job vacancies, see Shimer (2005) and Hall (2005). In this section, we show that the wage elasticity with respect to labour productivity has rather different determinants in the two models. The wage elasticity in our model, according to (2.13), is given by

$$\varepsilon_{w,A} = \frac{1}{\mu} \frac{e}{\delta} \frac{\partial f}{\partial A} \quad (2.30)$$

where  $\mu$  is the share of wage in output,  $\mu = w/A$ . Expression (2.30) shows that in our model, the wage responds to a productivity shock if and only if the job finding rate responds to productivity. If the job finding rate increases after a positive productivity shock, then the relative value of being unemployed to a worker also increases. This reduces the potential loss of being fired if the worker chooses to shirk since it becomes easier to find a new job. As a result, the firm will raise the wage to re-balance the incentive that the worker chooses not to shirk.

The extent to which the wage responds to a change in the job finding rate depends on the disutility of effort and monitoring technology. A lower disutility of effort or a higher detection rate will lead to a smaller response of the wage to a change in job finding rate. This is because in both cases the worker's incentive to shirk is lower so the firm does not need to raise the wage as much.

The wage elasticity in the standard DMP model, according to (2.29), is given by

$$\varepsilon_{w,A} = \frac{1}{\mu} (\phi + \phi\gamma' \frac{\partial \theta}{\partial A}) \quad (2.31)$$

Expression (2.31) shows that in the standard DMP model, the wage responds to a productivity shock for two reasons. One is that productivity affects the wage directly since productivity affects the size of the match surplus. The other is the response of labour market tightness to the productivity shock. A positive productivity shock makes job creation more ap-

pealing to the firms. So more job vacancies are posted. This pulls down the unemployment rate and increases labour market tightness. The increase in labour market tightness shortens unemployment duration, raising the value of being unemployed to the worker, and therefore raising worker's threat point in wage bargaining.

The extent to which the wage responds to changes in productivity and labour market tightness depends on the worker's bargaining power. A larger bargaining power implies that the worker will secure a larger share of any increase in productivity and have a higher threat point in wage bargaining. Expression (2.31) also says a larger vacancy cost will raise the worker's threat point in wage bargaining. This is because a larger vacancy cost lowers the firm's threat point in wage bargaining since walking away from the wage negotiation becomes more costly to the firm.

The different driving forces behind the wage elasticities in (2.30) and (2.31) make it difficult to say in which model the wage is more volatile. Using U.S data between 1951 and 2003, Shimer (2005) finds that the standard deviation of the job finding rate is much smaller than the standard deviation of labour market tightness. This may suggest the job finding rate is less responsive to productivity shock than labour market tightness. This finding is consistent with the implication of constant-return-to-scale of job matching in the standard DMP model. From (2.10), we have

$$f = m\theta^{1-\alpha} \quad (2.32)$$

Take the first order derivative of (2.32) with respect to the labour market tightness,

$$\frac{\partial f}{\partial \theta} = m(1 - \alpha)\theta^{-\alpha} \quad (2.33)$$

Under reasonable calibrations, this derivative is always less than one. This implies

$$\frac{\partial f}{\partial A} < \frac{\partial \theta}{\partial A} \quad (2.34)$$

So the driving force of the wage elasticity in (2.30) is less volatile than in (2.31). Later we show that, with a reasonable calibration strategy, this helps

our model better to capture the business cycle features of unemployment and job vacancies.

## 2.6 The Elasticity of Labour Market Tightness

Large fluctuations of labour market tightness in response to productivity shocks is a key feature of empirical labour market dynamics. In this section, we give an analysis of the determinants of the elasticity of labour market tightness in equilibrium. Combine the wage equation (2.14) and the job creation condition (2.17) and in steady-state we obtain

$$b + \chi + e\left[1 + \frac{r + \tau + f(\theta)}{\delta}\right] + \frac{(r + \tau)\gamma}{q(\theta)} = A \quad (2.35)$$

Expression (2.35) determines the equilibrium value of  $\theta$ . After implicit differentiation of (2.35), we obtain the elasticity of labour market tightness with respect to productivity as

$$\eta_{\theta,A} = \frac{\delta A}{e(1 - \alpha)f(\theta) + \alpha\delta(A - w)} \quad (2.36)$$

**Proof.** Implicit differentiation yields

$$\frac{d\theta}{dA} = \frac{1}{[q(\theta) + \theta q'(\theta)]\frac{e}{\delta} - \frac{(r+\tau)\gamma q'(\theta)}{q(\theta)^2}} = \frac{1}{[1 + \frac{\theta q'(\theta)}{q(\theta)}]\frac{e}{\delta}q(\theta) - \frac{\theta q'(\theta)}{q(\theta)}\frac{(r+\tau)\gamma}{\theta q(\theta)}} \quad (2.37)$$

since we know that  $\theta q'(\theta)/q(\theta) = -\alpha$ , substitute this into (2.37)

$$\frac{d\theta}{dA} = \frac{1}{[1 - \alpha]\frac{e}{\delta}q(\theta) + \alpha\frac{(r+\tau)\gamma}{\theta q(\theta)}} \quad (2.38)$$

then substitute the job creation condition (2.17) into (2.38) to replace  $\frac{(r+\tau)\gamma}{\theta q(\theta)}$ ,

$$\frac{d\theta}{dA} = \frac{1}{[1 - \alpha]\frac{e}{\delta}q(\theta) + \alpha\frac{A-w}{\theta}} \quad (2.39)$$

re-arrange the equation, we obtain

$$\frac{d\theta}{dA} = \frac{\delta\theta}{e(1-\alpha)\theta q(\theta) + \alpha\delta(A-w)} \quad (2.40)$$

then we derive the elasticity of labour market tightness with respect to productivity as

$$\eta_{\theta,A} = \frac{\delta A}{e(1-\alpha)f(\theta) + \alpha\delta(A-w)} \quad (2.41)$$

■

Expression (2.36) shows the determinants of the elasticity of labour market tightness. Consider the extreme case where the disutility of effort is zero and the detection rate is equal to one, then (2.36) simplifies to,

$$\eta_{\theta,A} = \frac{A}{\alpha(A-w)} \quad (2.42)$$

In this extreme case, only the gap between output and the wage and the matching elasticity influence the elasticity of labour market tightness. The same result is obtained if Nash wage bargaining in the standard DMP model is replaced by a Hall-type sticky wage, see Hall (2005) and Ljungqvist and Sargent (2015).

## 2.7 Summary of Key Equations

We summarize the key equations of our model as follows. Two endogenous variables, the wage and labour market tightness, are determined by the wage equation (2.14) and the job creation condition (2.16). We pin down employment using the dynamic equation for employment (2.7). This makes our model comparable to the standard search and matching model. The only difference between the two is wage determination. The model in this paper assumes firms choose the wage to avoid shirking. Whereas the wage in the standard DMP model is determined by Nash bargaining between the firm and the worker. The key equations are listed below:

Job creation condition,

$$A_t - w_t - \lambda_t = 0 \quad (2.43)$$



The optimal wage set by the firm,

$$w_t = b + e(1 + \frac{r + \tau_t + f_t}{\delta}) \quad (2.44)$$

The cost of hiring a worker,

$$\lambda_t = \gamma \{ \frac{1}{q_t} - \beta(1 - \tau_t) E_t \frac{1}{q_{t+1}} \} \quad (2.45)$$

The dynamic equation for employment,

$$n_t = (1 - \tau_{t-1})n_{t-1} + h_t \quad (2.46)$$

The definition of unemployment,

$$u_t = 1 - n_t \quad (2.47)$$

The matching function,

$$h_t = m u_t^\alpha v_t^{1-\alpha} \quad (2.48)$$

Labour market tightness,

$$\theta_t = \frac{v_t}{u_t} \quad (2.49)$$

The vacancy-filling rate,

$$q_t = m \theta_t^{-\alpha} \quad (2.50)$$

The job-finding rate,

$$f_t = \theta_t q_t \quad (2.51)$$

### 2.7.1 Linearised Model

We obtain a linear approximation around the steady state using a first-order Taylor series expansion. Details are contained in the appendix. Let  $\hat{x}_t$  denote the percentage deviation of a variable  $X_t$  around its steady state and let  $\varepsilon_t^X$  denote the shock to variable  $X_t$ . Then the linearized form of the key equations above are:

$$\varepsilon_t^A = \omega_1 \hat{w}_t + (1 - \omega_1) \hat{\lambda}_t \quad (2.52)$$

$$\hat{w}_t = \frac{e\bar{f}}{\delta\bar{w}} \hat{f}_t + \frac{e\tau}{\delta\bar{w}} \varepsilon_t^\tau \quad (2.53)$$

$$\hat{\lambda}_t = -\lambda_1 \hat{q}_t + (1 - \lambda_1) \hat{q}_{t+1} + \lambda_2 \varepsilon_t^\tau \quad (2.54)$$

$$\hat{n}_t = (1 - \tau) \hat{n}_{t-1} + \tau \hat{h}_t - \tau \varepsilon_{t-1}^\tau \quad (2.55)$$

$$\hat{u}_t = \frac{-\bar{n}}{1 - \bar{n}} \hat{n}_t \quad (2.56)$$

$$\hat{h}_t = \alpha \hat{u}_t + (1 - \alpha) \hat{v}_t \quad (2.57)$$

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t \quad (2.58)$$

$$\hat{q}_t = -\alpha \hat{\theta}_t \quad (2.59)$$

$$\hat{f}_t = \hat{\theta}_t + \hat{q}_t \quad (2.60)$$

where  $\omega_1 = \frac{\bar{w}}{\bar{w} + \lambda}$ ,  $\lambda_1 = \frac{1}{1 - \beta(1 - \tau)}$ , and  $\lambda_2 = \frac{\beta\tau}{1 - \beta(1 - \tau)}$ .

## 3 Model Evaluation

### 3.1 Calibration

Our calibration strategy is summarised in Table 1. We normalize a time period to be one quarter, and set the risk-free interest rate to  $r = 0.012$ , equivalent to an annual risk-free interest rate of 0.048 (Shimer, 2005). Therefore the discount factor  $\beta$  is set to equal to 0.988. Following Shimer (2005) and Hall (2005), the job separation rate is set as  $\tau = 0.1$ , so on average 3.3 percent of employed workers exit employment every month. There is no consensus in literature on the calibrated values of the cost of posting a vacancy  $\gamma$  and the utility of leisure  $b$ . In the third chapter of this thesis, we argue that larger values of vacancy costs and the utility of leisure can both

generate larger unemployment fluctuations. Since our goal is to evaluate the role of efficiency wages in propagating labour market volatility, we adopt the lower value of the vacancy cost and the utility of leisure in the literature. Following Shimer (2005), we assume the vacancy cost  $\gamma$  is 0.213 and the utility of leisure  $b$  is 0.4. We normalise average productivity to be  $A = 1$ .

Table 1— Values of Calibrated Parameters		
Parameter	Interpretation	Calibrated Value
$\tau$	Exogenous Separation Rate	0.1
$\beta$	Discount Factor	0.99
$r$	Risk-Free Interest Rate	0.01
$A$	Labour productivity	1
$b$	Unemployment Benefit	0.4
$\gamma$	Vacancy Cost	0.213
$e$	Disutility of Effort	0.06
$\chi$	Utility of Leisure	0.43
$m$	Matching Coefficient	2.1
$\alpha$	Matching Elasticity	0.5
$\delta$	Detection Rate	0.94

We set the matching coefficient to  $m = 1.355$ , the same as in Shimer (2005). The matching elasticity  $\alpha$  is set to equal to 0.5, a standard value in search literature, see Pissarides and Petrongolo (2001).

To the best of our knowledge, there are no existing calibrations of the detection rate and disutility of effort. The value of those two parameters are chosen so that the model can match the average unemployment in the US, 5.8% (from BLS data:1948Q1 to 2014Q4), and average job finding rate of 0.55 per month (see Hagedorn and Mankoskii 2008). Doing so we obtained  $\delta = 0.945$  and  $e = 0.036$ . We assume firms only face small monitoring costs. So the probability of being caught if the worker shirks is 94.5%. The calibrated value of  $e$  suggests that the incentive workers choose to shirk is very low. We believe this is reasonable since the shirking model assumes

that workers put forth no effort once they choose to shirk. A high incentive to shirk will bring a huge welfare loss because firms must raise the wage to generate a high unemployment rate which leads to a high threat point to workers. The implied equilibrium unemployment rate and the job finding rate in our model match average U.S data well, see Table 2.

Table 2— Values of Endogenous Variables for Calibration

Parameter	Interpretation	U.S Data	Our Model
$u$	Unemployment Rate	0.058	0.063
$\theta$	Labour Market Tightness	0.539	0.504
$f$	Job Finding Rate	0.550	0.497

Note: Unemployment Rate is from BLS data, 1948Q1-2014Q4

Labour Market Tightness is from JOLTS, see Hall (2005)

Job Finding Rate is from Hagedorn and Manovskii (2008)

We assume that shocks to labour productivity and job separation follow the AR(1) processes,

$$\varepsilon_t^A = \psi \varepsilon_{t-1}^A + \delta^A \quad (3.1)$$

$$\varepsilon_t^\tau = \varphi \varepsilon_{t-1}^\tau + \delta^\tau \quad (3.2)$$

We set  $\psi = 0.878$ , s.d ( $\delta^A$ ) = 0.01, and  $\varphi = 0.733$ , s.d ( $\delta^\tau$ ) = 0.05, so both the quarterly autocorrelation and standard deviation of labour productivity and job separation match U.S data, see Table 3.

Table 3— Statistics of Labour Productivity and Job Separation

Statistics	Labour Productivity	Job Separation
Quarterly Autocorrelation	0.878	0.733
Standard Deviation	0.020	0.075

Data Source: Shimer(2005), U.S monthly data, 1951-2003

## 3.2 Simulation Results

### 3.2.1 Unemployment Decomposition

In our model, equilibrium unemployment can be decomposed into two components. One is due to search and matching frictions in the labour market; the other one is due to the efficiency wage set to avoid shirking. To decompose unemployment, we set the disutility of effort equal to zero so workers have no incentive to shirk. In this case, all the unemployment is frictional unemployment. The results are listed in the Table 4.

Table 4— Unemployment Decomposition		
Total Unemployment	Frictional Unemployment	High Wage Unemployment
0.06 (100%)	0.01 (16.7%)	0.05 (83.3%)

Table 4 shows frictional unemployment only accounts for 12.6% of total unemployment in our model. This is not surprising since when workers have no incentive to shirk, firms pay workers the lowest wage which amounts to the utility of leisure. This result, however, does not contradict the implications of the standard DMP model. Rather they are consistent. In the standard DMP model, unemployment arises for two reasons. One is search frictions; the other is the wage premium due to workers' bargaining power. Nash wage bargaining ensures that the real wage exceeds the marginal rate of substitution between consumption and leisure and thus explains the existence of involuntary unemployment. If workers are assumed to have no bargaining power, then the wage in the standard DMP model will decrease to the utility of leisure, the same as in our model. In both models, it is the high wage that accounts for most of equilibrium unemployment.

### 3.2.2 Volatility of Key Labour Market Variables

Table 5 describes the standard deviation of key labour market variables from U.S data and simulation results of the standard DMP model and our model. The results in the second and the third column are from Shimer (2005). The data source for the second column is U.S monthly data from 1951 to 2003, for details see pp.27-34 of Shimer (2005).

Table 5— Labour Productivity and Separation Shocks				
	Standard Deviation of Key Labour Market Variables	U.S Data Shimer (2005)	Standard DMP Shimer (2005)	Our Model
$y$	Output	0.020	0.020	0.020
$s$	Job Separation	0.075	0.020	0.075
$u$	Unemployment Rate	0.190	0.031	0.189
$v$	Vacancy Rate	0.202	0.011	0.204
$\theta$	Market Tightness	0.382	0.037	0.368
$f$	Job Finding rate	0.118	0.014	0.184
$w$	Wage	—	—	0.018

The standard DMP column reports Shimer (2005)’s finding that the standard DMP model produces too little volatility of unemployment from realistic fluctuations in productivity and job separations. However in Shimer’s simulation, the standard deviation of job separations is less than one third of what the U.S data suggests. Comparing Shimer’s results in the third column in Table 5 and the third column in Table 6 in which only labour productivity shock is considered, we find that the separation shock has only a trivial effect on the volatility of labour market tightness.

The last column of Table 5 shows that the search and matching model with efficiency wages can replicate the observed volatility of unemployment, vacancy and labour market tightness well. In our model, the standard deviation of labour market tightness is almost 20 times as large as the standard deviation of average labour productivity. The standard deviation of unemployment and job vacancies are about 10 times as large as the standard

deviation of average labour productivity. All those results are also consistent with U.S data. The job finding rate in both models is more volatile than what the data indicates. This is probably because under the assumption of exogenous job separations, the separation shock generates a positive correlation between unemployment and vacancies, leading to labour market tightness being almost unchanged. Therefore the volatility of labour market tightness almost entirely relies on fluctuations in job creation. The wage is much less volatile than unemployment.

One might be concerned that the joint analysis of labour productivity and separation shocks may not say too much about the role of productivity shocks in unemployment and vacancy volatility. Next we carry out the simulation with only the productivity shock. Results are reported in Table 6. In the absence of the shock to job separation, the decrease in standard deviation of unemployment, job vacancies and labour market tightness in our model is trivial. This implies that the productivity shock is the driving force of labour market volatility.

Table 6— Labour Productivity Shocks

Standard Deviation of Key Labour Market Variables		U.S Data Shimer (2005)	Standard DMP Model Shimer (2005)	Our Model
$y$	Output	0.020	0.020	0.020
$s$	Job Separation	0.075	—	—
$u$	Unemployment Rate	0.190	0.009	0.172
$v$	Vacancy Rate	0.202	0.027	0.195
$\theta$	Market Tightness	0.382	0.035	0.367
$f$	Job Finding rate	0.118	0.01	0.184
$w$	Wage	—	—	0.018

### 3.2.3 Explaining the Volatility Results

Since unemployment, vacancies and the job-finding rate can be expressed as functions of labour market tightness (eg Shimer, 2005), the elasticity of

labour market tightness with respect to productivity shocks is at the core of the unemployment volatility puzzle. Considering the steady-state of our model, we can express the firm's optimality condition in (2.17) as

$$y = w + \lambda \quad (3.3)$$

where  $\lambda = \frac{\gamma(r+\tau)}{q}$ . This implies

$$\frac{\partial y}{\partial \theta} = \frac{\partial w}{\partial \theta} + \frac{\partial \lambda}{\partial \theta} \quad (3.4)$$

Using (2.14),  $\frac{\partial w}{\partial \theta} = \frac{e}{d} \frac{\partial f}{\partial \theta}$  or, since  $f = \theta q$ ,  $\frac{\partial w}{\partial \theta} = \frac{e}{d} q (1 + \frac{\theta}{q} \frac{\partial q}{\partial \theta})$ . Using (2.9), this simplifies to

$$\frac{\partial w}{\partial \theta} = (1 - \alpha) \frac{e}{d} \frac{f}{\theta} \quad (3.5)$$

From (3.3),  $\frac{\delta \lambda}{\delta \theta} = -(r + \tau) \gamma \frac{1}{\theta q} (\frac{\theta}{q} \frac{\partial q}{\partial \theta})$ , or

$$\frac{\partial \lambda}{\partial \theta} = \frac{\alpha(r + \tau) \gamma}{\theta q} \quad (3.6)$$

Combining (3.4)-(3.6), we obtain a simple expression for the elasticity of labour market tightness with respect to productivity shocks in steady-state:

$$\frac{\partial \theta}{\partial y} \frac{y}{\theta} = \frac{1}{(1 - \alpha) \frac{e}{d} f + \frac{\alpha(r + \tau) \gamma}{q}} \quad (3.7)$$

Using our parameter values and the average values of  $q$  and  $f$  from Table 2), the elasticity of labour market tightness with respect to the productivity shock is 17.89. This is close to the ratio of the volatility of labour market tightness to the volatility of productivity shocks in Table 2).

Using (3.3), we can express the elasticity as

$$\frac{\partial \theta}{\partial y} \frac{y}{\theta} = \frac{1}{(1 - \alpha) \frac{e}{d} f + \alpha(y - w)} \quad (3.8)$$

Equation (3.8) shows that our model is able to generate a large volatility of labour market tightness because of a small rate of profit,  $(y - w)$ , and



a small value of  $\frac{e}{d}$ . This is consistent with arguments in Hagedorn and Manovskii (2008) that a large volatility of labour market tightness requires a low rate of profit. Low profits induce firms to put relatively few resources into recruiting, leading to a low level of labour market tightness and a high vacancy filling rate. This in turn implies that variations in vacancies in response to productivity shocks are transmitted strongly into variations in unemployment.

The comparison with the standard search frictions model is useful here. Using (2.29) to write

$$\frac{\partial w}{\partial \theta} = \phi \frac{\partial y}{\partial \theta} + \phi \gamma \quad (3.9)$$

and using (3.3)-(3.4) and (3.6), we obtain

$$\frac{\partial \theta}{\partial y} \frac{y}{\theta} = \frac{1}{\frac{\phi}{(1-\phi)} \gamma \theta + \frac{\alpha(r+\tau)\gamma}{(1-\phi)q}} \quad (3.10)$$

Using (2.29), this is

$$\frac{\partial \theta}{\partial y} \frac{y}{\theta} = \frac{1}{(1-\alpha) \frac{\phi}{(1-\phi)} \gamma \theta + \alpha(1-b-\chi)} \quad (3.11)$$

Using the calibrated parameters in Table 1) and following Shimer (2005) in assuming  $\phi = 0.72$ , the elasticity of labour market tightness with respect to the productivity shock is 4.48; if we follow most of the existing literature and assume  $\phi = 0.5$ , the elasticity is 1.74. Both values are considerably smaller than the volatility observed in the data. As noted by Hagedorn and Makovskii (2008), the standard search frictions model can only match empirical volatilities by making the implausible assumptions that workers have very little bargaining power and that the value of leisure is large.

Ljungqvist and Sargent (2016) argue that all proposed solutions to the unemployment volatility puzzle require a small value for the "fundamental surplus", the "upper bound on the fraction of a job's output that the invisible hand can allocate to vacancy creation"; this is equivalent to the lowest value of the wage that is consistent with (3.3). Ljungqvist and Sargent (2016) express the elasticity of labour market tightness with respect to the

productivity shock as

$$\frac{\partial \theta}{\partial y} \frac{y}{\theta} = \Gamma \frac{y}{y - \xi} \quad (3.12)$$

where  $\xi$  is the fundamental surplus and  $\frac{y-\xi}{y}$  is the fundamental surplus share. In our model, the no-shirking constraint gives the lowest wage that is consistent with positive output, so  $\xi = w$  and so the fundamental surplus share is simply the rate of profit. We can show that  $\Gamma = \frac{(r+\tau)\gamma\theta}{\alpha(r+\tau)\gamma+(1-\alpha)\frac{e}{d}fq}$ . As with (3.11), the low rate of profit in our model generates a large volatility of labour market tightness. In the standard search frictions model, the fundamental surplus is  $\xi = b + \chi$ , as this is the lowest value of the wage that is consistent with non-zero output. In this case,  $\Gamma = \frac{(r+\tau)+\phi f}{\alpha(r+\tau)+\phi f}$ . The inability of the standard search frictions model to address the volatility puzzle is reflected in the relatively small values of  $\Gamma$  and  $\xi$  obtained by Ljungqvist and Sargent (2016) using standard calibrations. The value of  $\frac{y}{y-\xi}$  is only large enough to generate substantial volatility in  $\theta$  when the value of  $b + \chi$  is assumed to be large, following Hagedorn and Makovskii (2008). In the case of the strategic bargaining model of Hall and Milgrom (2008) (see also Christiano et al, 2016), the fundamental surplus is  $\xi = b + \chi + \frac{(1-\tau)}{1+r}\gamma$ , where  $\gamma$  is the fixed cost of delay incurred by the firm; in this case,  $\Gamma = \frac{1}{\alpha}$ . Ljungqvist and Sargent (2016) argue that a large cost of delay is required to generate a large volatility of labour market tightness.

### 3.2.4 Variance Decomposition

Table 7 shows the variance decomposition of the key labour market variables when the labour productivity shock and the job separation shock are jointly taken into account. In our model, the separation shock plays a non-trivial role in terms of generating unemployment volatility, which is the same as in Shimer (2005). However for each labour market variable the productivity shock is unarguably the major driving force behind their volatilities.

Table 7— Variance Decomposition

	Key Labour	Productivity Shock	Separation Shock
Market Variables			
$u$	Unemployment Rate	82.76	17.24
$v$	Vacancy Rate	91.10	8.90
$\theta$	Market Tightness	99.77	0.23
$f$	Job Finding rate	99.77	0.23
$w$	Wage	99.96	0.04

### 3.2.5 Autocorrelation and Cross Correlations

The model in the paper also outperforms the standard DMP model in terms of replicating the autocorrelation of job vacancies and the correlation between unemployment, job vacancies, labour market tightness and the job finding rate.

The first column of the Table 8 shows the quarterly autocorrelations of key labour market variables observed in the data. The second column reports the same statistics obtained by simulating the standard DMP model. The quarterly autocorrelation of job vacancies in the DMP model is less than one third of its counterpart in the data. By contrast, the same autocorrelation in our model is much closer to the data. This might indicate that our models can better reflect the dynamics of job creation.

Table 8— Quarterly Autocorrelation of Key Labour Market Variables

	Key Labour	U.S Data	Standard DMP	Our Model
	Market Variables	Shimer (2005)	Shimer (2005)	
$u$	Unemployment Rate	0.936	0.933	0.853
$v$	Vacancy Rate	0.940	0.291	0.866
$\theta$	Market Tightness	0.941	0.878	0.878
$f$	Job Finding rate	0.908	0.878	0.878
$y$	Output	0.878	0.878	0.878
$s$	Job Separation	0.733	0.878	0.733

Table 9 shows the correlation matrix of key labour market variables in each model. U.S data indicates the correlation between unemployment and job vacancies is -0.894, so the two variables are strongly negatively correlated. However the standard DMP model cannot capture this feature, the correlation between two variables in the DMP model is only -0.427. Our model does a better job on this. The correlation between unemployment and job vacancies in our model is -0.768. The standard DMP model also cannot capture the observed correlation between job vacancies and labour market tightness and the observed correlation between job vacancies and job finding rate. Here, again, our model outperforms the standard DMP model. This may reflect that our search models with efficiency wage better capture the labour market dynamics.

Table 9— Matrix of Correlation of Key Labour Market Variables

		$u$	$v$	$\theta$	$f$
U.S Data	$u$	1	-0.894	-0.971	-0.949
Standard DMP	$u$	1	-0.427	-0.964	-0.964
Our Model	$u$	1	-0.744	-0.929	-0.929
U.S Data	$v$	—	1	0.975	0.897
Standard DMP	$v$	—	1	0.650	0.650
Our Model	$v$	—	1	0.939	0.939
U.S Data	$\theta$	—	—	1	0.948
Standard DMP	$\theta$	—	—	1	1
Our Model	$\theta$	—	—	1	1
U.S Data	$f$	—	—	—	1
Standard DMP	$f$	—	—	—	1
Our Model	$f$	—	—	—	1

### 3.2.5 Impulse Responses

Figure 1 displays the dynamic responses of nine macro variables (employment, labour market tightness, wage, unemployment, hiring costs, vacancy-filling rate, job vacancies, job-finding rate and aggregate hires) to an ex-

ogenous positive productivity shock, using the search frictions model with efficiency wages. All the responses are in the right direction. The responses of labour market tightness, unemployment and the job vacancy to the productivity shock are larger than the response of the wage to the productivity shock. This is consistent with our findings in Table 6.

Figure 2 displays the corresponding responses to an exogenous positive job separation shock. It shows that both unemployment and job vacancies increase in response to a higher job separation rate. The increase in unemployment is slightly larger than the increase in job vacancies. This leads to a small decrease in labour market tightness. Comparing with Figure 1, we see that the responses to the separation shock are less persistent than to the productivity shock.

## 4 Conclusions

In this chapter we have added a standard shirking mechanism to an otherwise standard search frictions model. We have found that the wage in our model exhibits relatively little cyclical variation. We also found that this alternative type of search frictions model can deliver the same unemployment and job vacancies fluctuations as in U.S data. The model also matches labour market dynamics well.

The findings in the first two chapters have interesting implications. We show that efficiency wage theory can explain why wages are less responsive to productivity shocks when this hypothesis is incorporated into a dynamic labour market with search frictions. When a Solow-type efficiency wage model is considered, the cyclicity of the wage depends on the cyclicity of the reference wage of workers. We give two alternative ways to define the reference wage. One is based on the idea of social norms and fair wages. The other one is based on the expected return of outside option. In the second approach, under rational expectations, workers realise that a temporary technology shock only has a limited effect on the expected return of the outside option which is measured based on their expected life-time career path. So unless workers are ‘myopic’, workers know even if they successfully

switch to another job which comes with a higher payment when a positive productivity shock is observed, such a relatively high payment cannot last long since the shock is temporary. As a result, they prefer to stay in the current job to receive a stable income although their employers do not increase the wage much. This microfounded reference wage plays a key role in delivering the little cyclical variation of the wage.

Another even more microfounded approach has been explored in this chapter. Based on Shapiro and Stiglitz (1984), the wage is chosen to avoid shirking. A positive productivity shock increases the worker's incentive to shirk if and only if this positive productivity shock can increase the probability of finding a new job. Empirical evidence shows that job finding rate is not as volatile as other labour market variables such as labour market tightness. This provides another way to explain the mild cyclical variation of the wage.

A common finding in the two chapters is that under exogenous job separations the separation shock generates a positive correlation between unemployment and vacancies, leading to the labour market tightness being almost unchanged. Therefore the volatility of labour market tightness almost entirely relies on the fluctuations of job creation. This is consistent with Shimer (2005). However given the fact that our three models capture the volatility of unemployment, job vacancies and labour market tightness well but the job finding rate is much more volatile than U.S data, we believe this suggests that job separation might be an important driving force of the labour market volatility. We turn to this topic in the last chapter of the thesis.

## APPENDIX OF CHAPTER 2

### The Linear Approximation

*The optimal wage*

$$w_t = b + e(1 + \frac{r + \tau_t + f_t}{\delta})$$

One can write this as

$$\ln w_t = \ln[b + e(1 + \frac{r + \tau_t + f_t}{\delta})]$$

then rewrite each variable as its first-order taylor polynomial at the steady state

$$\ln w + \frac{1}{w}(w_t - w) = \ln[b + e(1 + \frac{r + \tau + f}{\delta})] + \frac{e}{w\delta}(f_t - f) + \frac{e}{w\delta}(\tau_t - \tau)$$

since  $\ln w = \ln[b + e(1 + \frac{r+\tau+f}{\delta})]$ , it follows that

$$\frac{1}{w}(w_t - w) = \frac{e}{w\delta}(f_t - f) + \frac{e}{w\delta}(\tau_t - \tau)$$

Using  $\hat{x}_t$  to denote the percentage deviation of a variable  $X_t$  around its steady state, this becomes

$$\hat{w}_t = \frac{e\bar{f}}{\delta\bar{w}}\hat{f}_t + \frac{e\tau}{\delta\bar{w}}\varepsilon_t^\tau$$

See the appendix of the previous chapter for the linear approximation of the rest of equations.

Figure 1: Impulse Responses of Key Labour Market Variables to Productivity Shock

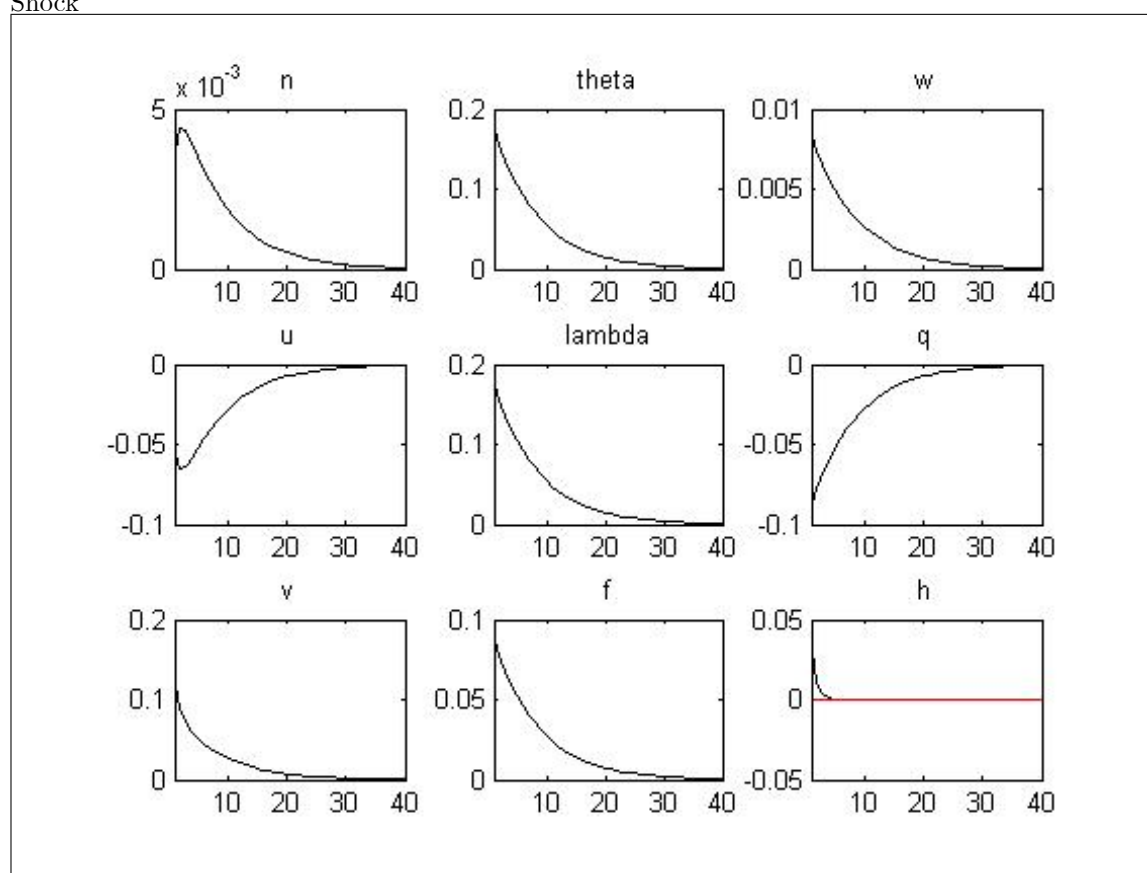
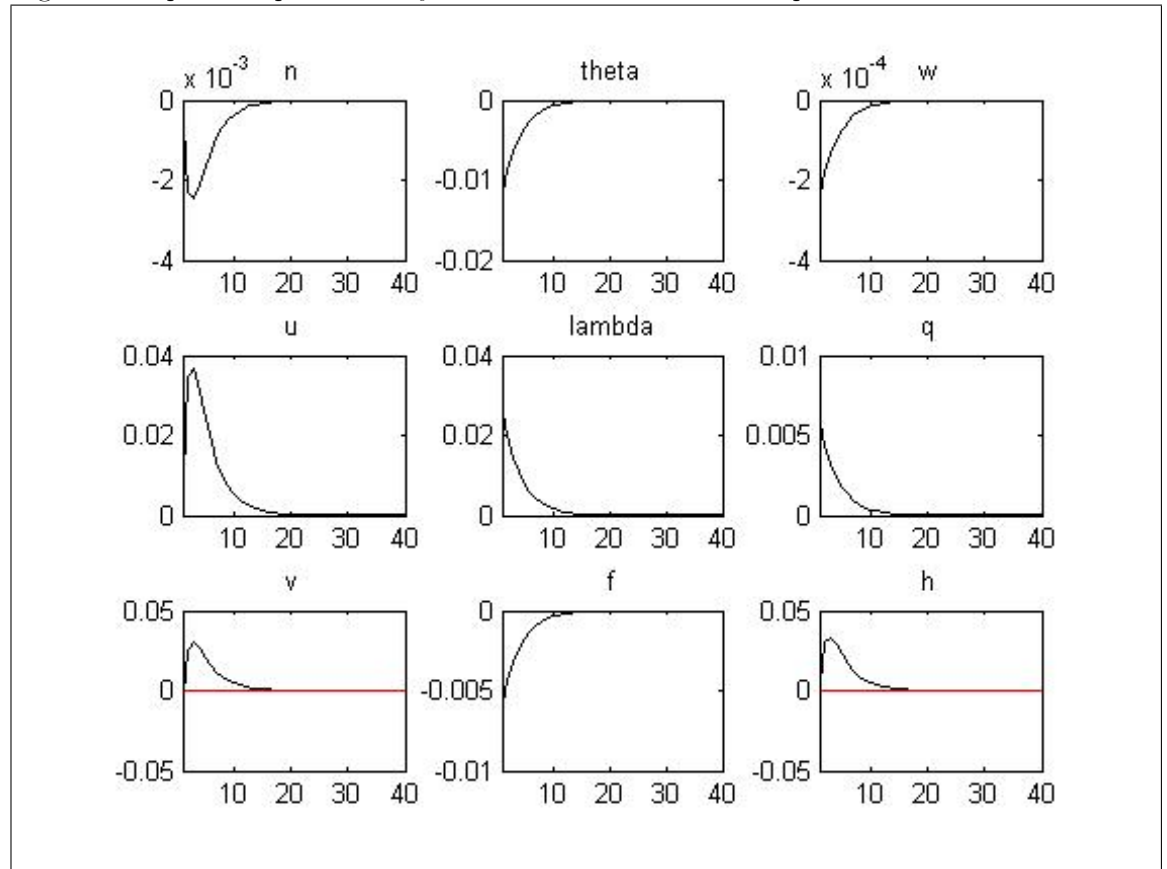




Figure 2: Impulse Responses of Key Labour Market Variables to Separation Shock



## Chapter 3

# The Two Solutions to the Unemployment Volatility Puzzle

### Abstract

This chapter analyses two proposed solutions to the unemployment volatility puzzle: sticky wages and a small ‘hiring surplus’. We argue that sticky wages ensures that job vacancies vary with labour productivity and a small ‘hiring surplus’ ensures any variation of job vacancies is more likely to be transmitted to variation in unemployment. We point out that a widely used calibration strategy in the literature raises the relative cost of creating a job vacancy and increases matching efficiency. We argue that this calibration strategy is a key factor in generating a large unemployment volatility.

# 1 Introduction

The Diamond-Mortensen-Pissarides (DMP) search and matching model gives an appealing description of how unemployment arises in equilibrium and what makes it change over time. However, as widely known, the model is incapable of matching the observed fluctuations in unemployment and job vacancies, which is often referred as the unemployment volatility puzzle. The early literature argued that this is because the Nash bargained wage in the DMP model is too flexible in the sense that the wage absorbs most of the productivity changes caused by a shock. The causal chain behind this argument is straightforward. An increase in labour productivity makes job creation more appealing to the firms and so more job vacancies are posted. The increase in job vacancies shortens unemployment duration, raising the value of being unemployed to the worker. This raises the worker's threat point in wage bargaining. A higher labour productivity also increases the match surplus, raising the share of the surplus to the worker. Those two effects lead to higher wages. As a result, wages absorb most of the productivity increase, eliminating the firms' incentive for vacancy creation.

This argument triggered a wide discussion on modelling alternative wage regimes<sup>11</sup>, see for example, Hall (2005), Hall and Milgrom (2008), Kennan (2009), Rudanko (2009), Michaillat (2012) and Rudanko and Krusell (2015)<sup>12</sup>. The common feature of those alternative wage regimes is that the wage is less responsive to labour productivity changes. We refer this common feature as the 'sticky wage' in the following discussion.<sup>13</sup>

The literature also discusses alternative scenarios on the unemployment

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<sup>11</sup>As emphasized by Hall (2005), an existing firm-worker pair will be privately efficient so long as it generates a positive surplus to both parties involved. Any wage path that can guarantee this private efficiency is consistent with equilibrium.

<sup>12</sup>The literature also incorporates the DMP framework with sticky wages into the DSGE model to study the effect of the productivity shocks on unemployment, see Gertler, Sala and Trigari (2008), Thomas (2008), Gali (2011) and Christiano, Eichenbaum and Trabandt (2015).

<sup>13</sup>We argue that the Nash bargained wage also displays some stickiness since the wage cannot absorb all of the productivity change unless the worker has full bargaining power. However in that case, there will be no hire.

volatility puzzle. For example, Hagedorn and Manovskii (2008) controversially raise the value of leisure when they calibrate the standard DMP model; Pissarides (2009) introduces fixed matching costs to the DMP model; Petrosky-Nadeau and Wasmer (2013) embed credit frictions into the DMP model.

We point out a common channel behind those alternative scenarios. This common channel works through a high rate of filling a vacancy. A high rate of filling a vacancy makes any newly posted vacancy due to a positive productivity shock more likely to be filled and makes any reduction of job vacancies due to a negative productivity shock more likely to be transmitted to job losses. This generates larger fluctuations in unemployment. We argue that to achieve a high vacancy-filling rate, those scenarios diminish the firm's benefit from hiring a worker<sup>14</sup>. This leads to low incentives for vacancy creation and causes increased slack in the labour market, which in turn shortens the duration of a vacancy and increases the vacancy-filling rate. We refer to this common channel as the 'small hiring surplus' in the following discussion.

We argue that the sticky wage and the 'small hiring surplus' act as complements rather than substitutes. The sticky wage ensures that job vacancies vary with labour productivity. The 'small hiring surplus' ensures that any variation in job vacancies is more likely to be transmitted to variation in unemployment.

The points we have made so far are also a critical response to a recent paper by Ljungqvist and Sargent (2015). They define the upper bound of resources available for vacancy creation as the 'fundamental surplus' for the firm. And they argue that a small 'fundamental surplus' is the 'single common channel' to solve the unemployment volatility puzzle.

Our argument is different to the 'small fundamental surplus' in the following respects. First, we believe that the sticky wage is a distinct channel and a necessary condition for generating unemployment fluctuations. Whereas Ljungqvist and Sargent (2015) didn't highlight the role of sticky wage in

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<sup>14</sup>In standard DMP model, each firm hires one worker. So the benefit and the marginal benefit of hiring a worker are the same.

generating unemployment fluctuations. Rather, they simply argue that a ‘low elasticity of the wage with respect to productivity’ is ‘neither a necessary nor a sufficient condition’. We agree that a low wage elasticity is not a necessary condition for generating unemployment fluctuations. However the wage has to display some stickiness in the sense that the wage cannot absorb all the productivity change. Suppose a wage regime allows the wage to move proportionally with labour productivity but its level is slightly smaller than labour productivity. This wage regime satisfies the private efficiency condition since it generates a positive surplus to both the firm and the worker. It also leads to a small ‘fundamental surplus’. However, since this wage regime implies a constant ‘hiring surplus’, there would be no response of job vacancies and unemployment to the productivity shock.

Second, we argue that a small ‘hiring surplus’ is more transparent than a small ‘fundamental surplus’ for two reasons. One is that we lay out the transmission mechanism behind the ‘small hiring surplus’. Whereas Ljungqvist and Sargent (2015) only explain why the small ‘fundamental surplus’ matters. It is difficult to see how the small ‘fundamental surplus’ works through the ‘invisible hand’. Secondly, the mathematical expression for the ‘hiring surplus’ is the same for different models since it is simply the hiring cost which the firm saves from an existing job match. By contrast the mathematical expression for the ‘fundamental surplus’ varies with the model specification<sup>15</sup>.

We find that a widely used calibration strategy in the literature implies a large unemployment volatility. Under this calibration strategy, firms face a relatively large cost of posting a vacancy, see for example, Hall (2005), Hagedorn and Manovskii (2008) and Hall and Milgrom (2008). This common feature is obscured by different time periods used in calibrations in the literature. We adjust the calibrations used in different models to make them all correspond to a monthly frequency. It turns out that this calibration implies a high vacancy posting cost is associated with low job vacancies and

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<sup>15</sup>In some cases, it is impossible to derive an expression for the ‘fundamental surplus’. For instance, the DMP model with endogenous job separation or the DMP model with financial frictions in Petrosky-Nadeau and Wasmer (2013).

high vacancy-filling rate since a relatively large vacancy cost reduces vacancy creation, giving rise to increased labour market slack.

Low job vacancies in equilibrium will lead to a higher equilibrium unemployment rate, given the parameters of the matching function. However a common target for calibrating search models is to match the average unemployment rate in U.S. This requires higher matching efficiency in the sense that fewer vacancies are required to form a job match. We calculate the number of vacancies needed to maintain an equilibrium unemployment rate, based on the matching functions in different literatures. This confirms our argument that the models in the literature that have lower job vacancies in equilibrium assume higher matching efficiency in order that the models are able to match observed average unemployment rates.

We further argue that a higher matching efficiency also increases the vacancy-filling rate, given the wage regimes in the literature. Therefore this calibration strategy implies a large unemployment volatility.

The chapter is organized as follows. Section 2 outlines the basic equations of the DMP model. Section 3 shows how the sticky wage and the ‘small hiring surplus’ act as complements in terms of generating large unemployment volatility. Section 4 gives a comparison between our arguments and the ‘fundamental surplus’ approach of Ljungqvist and Sargent (2015). Section 5 and 6 jointly investigate the impact of the common calibration strategy on the unemployment volatility. Section 7 concludes.

## 2 Search and Matching Model

To set the stage, we review the key equations and equilibrium relationships for a basic continuous time DMP model. There is a continuum of identical individuals on the unit interval. Each individual inelastically supplies one unit of labour and consumes all the income they earn. They are infinitely lived and risk neutral with discount rate denoted by  $r$ . An individual is either employed and earning a wage  $w$ , or else unemployed and enjoy the utility of leisure  $b$ . If unemployed, an individual finds a job with probability  $f$ . We assume that existing job matches are terminated with probability  $\tau$ .

Since we assume that all firms are identical, the value of being employed is thus

$$rL = w + \tau(U - L) \quad (2.1.1)$$

while the value of being unemployed is

$$rU = b + f(L - U) \quad (2.1.2)$$

There is a continuum of identical firms on the unit interval. Each firm can hire up to one worker who produces an amount  $y$ . Existing jobs command rents  $y - w$  for the firm in equilibrium. The value of a filled job is

$$rJ = y - w - \tau J \quad (2.1.3)$$

Firms must pay a real fixed cost of  $c$  to post a vacancy. Vacancies are then filled with probability  $q$ ; if the vacancy is filled, the new job match becomes productive in the following period. The value of a vacant job is then

$$rV = -c + q(J - V) \quad (2.1.4)$$

Trade in the labour market is uncoordinated, time-consuming, and costly for both firms and workers. The number of jobs formed in each period is determined by a matching function  $M(u, v)$  where  $u$  is the number of job seekers and  $v$  are aggregate vacancies. The matching function  $M(u, v)$  is increasing in both arguments, concave and homogeneous of degree one. We proceed under the assumption that the matching function has the Cobb-Douglas form,  $h = mu^\alpha v^{1-\alpha}$  where  $m > 0$  and  $\alpha$  is the constant elasticity of matching with respect to unemployment. Defining  $\theta = \frac{v}{u}$  as a measure of the tightness of the labour market, total hiring  $h$  is

$$h = mu\theta^{1-\alpha} \quad (2.1.5)$$

The probability of a firm filling a vacancy is

$$q = \frac{h}{v} = m\theta^{-\alpha} \quad (2.1.6)$$

While the probability that an unemployed worker finds a job is

$$f = \frac{h}{u} = \theta q(\theta) \quad (2.1.7)$$

The homogeneity of the matching function implies  $q(\theta)' < 0$ ,  $q(\theta)'' > 0$  and  $f(\theta)' > 0$ ,  $f(\theta)'' < 0$ . It is worth noting that  $\alpha = -q'(\theta)\theta/q(\theta)$ .

The number of workers who enter unemployment in each period is  $\tau(1 - u)$ , and the number who leave unemployment is  $\theta q(\theta)u$ . In equilibrium, unemployment is constant, so the two flows are equal,  $\tau(1 - u) = \theta q(\theta)u$ . I rewrite unemployment as the function of two transition rates:

$$u = \frac{\tau}{\tau + \theta q(\theta)} \quad (2.1.8)$$

Equation (2.1.8) is the first key equation of the model. It is known as the *Beveridge curve* when represented in vacancy-unemployment space. In equilibrium, free entry in the labour market drives rents from vacant jobs to zero. Therefore the equilibrium condition for vacancy creation is  $V = 0$ , implying that

$$J = \frac{c}{\beta q(\theta)} \quad (2.1.9)$$

Condition (2.1.9) states that in equilibrium, the expected profit from a new job is equal to the expected cost of hiring a worker. Substitute equilibrium condition (2.1.9) into (2.1.3),

$$y - w - \frac{(r + \tau)c}{q(\theta)} = 0 \quad (2.1.10)$$

Equation (2.1.10) describes the marginal condition for the demand for labour. Total output  $y$  is used to pay the wage  $w$  and the expected capitalized value of the firm's hiring cost  $\frac{(r+\tau)c}{q(\theta)}$ .



## 3 Sticky Wages and the Hiring Surplus

### 3.1 Unemployment Fluctuations

Define the firm's benefit of hiring a worker,  $y - w$ , as the 'hiring surplus', denoted by  $\lambda$ . From (2.1.10), we have

$$\lambda = \frac{(r + \tau)c}{q(\theta)} \quad (3.1.1)$$

Rewrite equation (3.1.1) as a function for  $q(\theta)$

$$q(\theta) = \frac{(r + \tau)c}{\lambda} \quad (3.1.2)$$

Equation (3.1.2) shows that a small 'hiring surplus' is associated with a high rate of filling a job vacancy. Substitute (2.1.6) into (3.1.2) to replace  $q(\theta)$  and solve for  $\theta$ ,

$$\theta = \left[ \frac{m\lambda}{(r + \tau)c} \right]^{\frac{1}{\alpha}} \quad (3.1.3)$$

Equation (3.1.3) shows a positive correlation between the 'hiring surplus' and labour market tightness. A small 'hiring surplus' leads to firms having a low incentive for vacancy creation, giving rise to low labour market tightness. A small 'hiring surplus' can be caused by high unemployment compensation as in Hagedorn and Manovskii (2008); a fixed matching cost as in Pissarides (2009) or credit frictions as in Petrosky-Nadeau and Wasmer (2013).

Recall that aggregate hires are defined as

$$h = vq(\theta) \quad (3.1.4)$$

Using the definition of labour market tightness, write  $v$  as

$$v = \theta u \quad (3.1.5)$$

then using (2.1.8) to replace  $u$  in (3.1.5),

$$v = \frac{\tau\theta}{\tau + \theta q(\theta)} \quad (3.1.6)$$

substituting (3.1.6) into (3.1.4) to replace  $v$ ,

$$h = \frac{\tau \theta q(\theta)}{\tau + \theta q(\theta)} \quad (3.1.7)$$

then substituting (3.1.2) and (3.1.3) into (3.1.7) so we can write  $h$  as an explicit function of  $\lambda$ ,

$$h(\lambda) = \frac{\tau m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1}}{\tau + m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1}} \quad (3.1.8)$$

This is the first key equation in this section. For the moment, we use  $h(\lambda)$  to denote (3.1.8). Given a change in  $\lambda$ , the movement in aggregate hires is given by

$$h(\dot{\lambda}) = h'(\lambda) \dot{\lambda} \quad (3.1.9)$$

where  $h'(\lambda) > 0$ . We assume that the change in  $\lambda$  is caused by productivity change. Rewrite (2.1.10) as

$$\lambda = y - w \quad (3.1.10)$$

Taking the derivative of (3.1.10) with respect to  $y$ , we have

$$\frac{\partial \lambda}{\partial y} = 1 - \frac{\partial w}{\partial y} \quad (3.1.11)$$

This is the second key equation in this section. It shows that the change in  $\lambda$  depends on the wage's response to the productivity change. Using (3.1.11) to rewrite  $\dot{\lambda}$  as a function of  $\dot{y}$  and substitute this into (3.1.9),

$$h(\dot{\lambda}) = h'(\lambda) \cdot \left( 1 - \frac{\partial w}{\partial y} \right) \dot{y} \quad (3.1.12)$$

Recall that in equilibrium, labour inflows to employment pool are equal to labour outflows

$$h(\lambda) = \tau(1 - u) \quad (3.1.13)$$

Taking the first-order difference of (3.1.13),

$$\dot{u} = -\frac{1}{\tau}h(\dot{\lambda}) \quad (3.1.14)$$

Substitute (3.1.12) into (3.1.14), and we have the following proposition

**Proposition 1** *In a labour market with search and matching frictions and exogenous separation rate  $\tau$ , an individual firm pays  $\lambda$  to hire a worker and the free entry condition ensures  $y = w + \lambda$ . For a given change in productivity,  $\dot{y}$ , the corresponding change in unemployment is determined by*

$$\dot{u} = -\frac{1}{\tau}h'(\lambda) \cdot (1 - \frac{\partial w}{\partial y})\dot{y} \quad (3.1.15)$$

where  $h'(\lambda) > 0$ .

## 3.2 Discussion

Proposition 1 shows the determinants of unemployment fluctuations. The first determinant  $h'(\lambda)$  captures the rate of change of aggregate hires with respect to the ‘hiring surplus’. The second determinant  $1 - \frac{\partial w}{\partial y}$  measures the change in the ‘hiring surplus’ given a unit change in productivity. The constant  $-\frac{1}{\tau}$  transfers fluctuations in aggregate hires to fluctuations in unemployment. Obviously, the change of the ‘hiring surplus’ depends on the response of the wage to the productivity change,  $\frac{\partial w}{\partial y}$ . If the wage absorbs all of the productivity change, then the ‘hiring surplus’ will not change. This would lead to no response of aggregate hires to the productivity change. A sticky wage thus ensures the ‘hiring surplus’ varies with productivity<sup>16</sup>.

To study the rate of change of aggregate hires with respect to the ‘hiring surplus’, we establish the following proposition

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<sup>16</sup>In a more complex set-up, the ‘hiring surplus’ can be defined as  $y - w - z$ , where  $z$  denotes other production costs. In this case, the change of the ‘hiring surplus’ can be written as  $1 - \frac{\partial w}{\partial y} - \frac{\partial z}{\partial y}$ . Normally the literature would assume this extra cost is insulated from the labour productivity, so  $\frac{\partial z}{\partial y} = 0$ . However whether or not this assumption holds,  $\frac{\partial w}{\partial y}$  still plays a key role in determining the change of the ‘hiring surplus’.

**Proposition 2** For given parameter value of  $c$ ,  $r$ , and  $\tau$ , if the elasticity of matching with respect to unemployment satisfies  $0 < \alpha < 1$  and if  $f > \tau$ , then we have  $h''(\lambda) < 0$ .

**Proof.** Rewrite the aggregate hires function (3.1.8) as

$$h(\lambda) = \tau - \frac{\tau^2}{\tau + m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1}} \quad (3.2.1)$$

Taking the first-order derivative of (3.2.1) with respect to the ‘hiring surplus’,

$$\frac{\partial h(\lambda)}{\partial \lambda} = \frac{\tau^2 \left( \frac{1}{\alpha} - 1 \right) m^{\frac{1}{\alpha}} \frac{1}{(r+\tau)c} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-2}}{\left( \tau + m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \right)^2} \quad (3.2.2)$$

Then taking the second-order derivative of (3.2.1) with respect to the ‘hiring surplus’,

$$\frac{\partial^2 h(\lambda)}{\partial^2 \lambda} = \frac{A \left\{ \left( \frac{1}{\alpha} - 2 \right) \left( \tau + m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \right) - 2 \left( \frac{1}{\alpha} - 1 \right) m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \right\}}{\left( \tau + m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \right)^4} \quad (3.2.3)$$

where  $A = \tau^2 \left( \frac{1}{\alpha} - 1 \right) m^{\frac{1}{\alpha}} \lambda^{\frac{1}{\alpha}-3} \left[ \frac{1}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \left( \tau + m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \right)$ . Since we assume that  $0 < \alpha < 1$ , we have  $A > 0$ .

Defining  $B = \left( \frac{1}{\alpha} - 2 \right) \left( \tau + m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \right) - 2 \left( \frac{1}{\alpha} - 1 \right) m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1}$ , rearranging the equation,

$$B = \left( \frac{1}{\alpha} - 2 \right) \tau - \frac{1}{\alpha} m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \quad (3.2.4)$$

To see whether  $B$  is a negative number, we consider two cases:

I. If  $\frac{1}{\alpha} - 2 \leq 0$ , then  $B$  is negative and therefore  $h''(\lambda) < 0$ .

II. If  $\frac{1}{\alpha} - 2 > 0$ , a necessary and sufficient condition to ensure  $h''(\lambda) < 0$  is

$$\left( \frac{1}{\alpha} - 2 \right) \tau < \frac{1}{\alpha} m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1} \quad (3.2.5)$$

Recall that  $f = m^{\frac{1}{\alpha}} \left[ \frac{\lambda}{(r+\tau)c} \right]^{\frac{1}{\alpha}-1}$ , substitute this into (3.2.5)

$$(\frac{1}{\alpha} - 2)\tau < \frac{1}{\alpha}f \quad (3.2.6)$$

Rearrange the condition (3.2.6),

$$f > (1 - 2\alpha)\tau \quad (3.2.7)$$

A sufficient condition to satisfy the condition (3.2.7) is  $f > \tau$ . Recall that in equilibrium, we have  $fu = \tau(1 - u)$ . Since  $u$  is unlikely larger than 0.5 in normal case, we have  $f > \tau$ . Therefore in both cases we have  $h''(\lambda) < 0$ . ■

The implication of proposition 2 is the following. To have a non-trivial response of aggregate hires to a given change in the ‘hiring surplus’, the ‘hiring surplus’ has to be small. The causal chain behind this argument is straightforward. A small ‘hiring surplus’ gives firms a low incentive for vacancy creation. This results in a slack labour market, which in turn shortens the duration of filling a vacancy and increases the vacancy-filling rate. A high vacancy-filling rate ensures that any variation in job vacancies is more likely transmitted to variation in aggregate hires.

Given proposition 2, the negative correlation between the ‘hiring surplus’ and the vacancy-filling rate in (3.1.1) leads to the following corollary.

**Corollary 3** *For any  $\lambda$  and  $\theta$ , we have  $\partial h'(\lambda)/\partial q(\theta) > 0$ .*

Corollary 3 confirms our argument that a high vacancy-filling rate leads to a large response of aggregate hires to a given change in the ‘hiring surplus’.

## 4 Comparison with the ‘Fundamental Surplus’

### 4.1 Definition

Ljungqvist and Sargent (2015) argue that a necessary and sufficient condition to generate large responses of unemployment to productivity changes is a small ‘fundamental surplus’, the upper bound on the fraction of output available for vacancy creation. To derive an expression for the ‘fundamental surplus’ in the standard DMP model with Nash Bargaining, we combine

the job creation condition and the wage equation to obtain an equilibrium condition for labour market tightness. Recall the wage equation under Nash bargaining<sup>17</sup>,

$$w = b + \phi(y - b + \theta c) \quad (4.1.1)$$

Substituting the wage equation into the job creation condition (2.1.10) gives

$$y - b = \frac{r + \tau + \phi\theta q(\theta)}{(1 - \phi)q(\theta)}c \quad (4.1.2)$$

After implicit differentiation of (4.1.2), we derive the elasticity of labour market tightness with respect to labour productivity<sup>18</sup>,

$$\varepsilon_{\theta,y} = \frac{r + \tau + \phi\theta q(\theta)}{\alpha(r + \tau) + \phi\theta q(\theta)} \frac{y}{y - b} \quad (4.1.3)$$

Ljungqvist and Sargent (2015) defines the ‘fundamental surplus’ as  $y - b$ , which is what remains after deducting the unemployment compensation (or say the value of leisure) from the output. Equation (4.1.3) shows that a large elasticity of labour market tightness with respect to productivity requires a small ‘fundamental surplus’.

## 4.2 Comparison

Ljungqvist and Sargent (2015) define the ‘fundamental surplus’ as the resources which the ‘invisible hand’ can allocate to vacancy creation. Whereas the ‘hiring surplus’ in this chapter is the resources an individual firm can allocate to vacancy creation. In the standard DMP model with Nash Bargaining, the ‘fundamental surplus’,  $y - b$ , is larger than the ‘hiring surplus’,  $y - w$ . This is simply because the ‘invisible hand’, or a benevolent dictator in a centralised economy, has more power over resource allocation. More specifically, a firm’s behavior is bound by the Nash sharing rule however the ‘invisible hand’ is not. In spite of this difference, there are still some linkages between the two. If we substitute the wage equation into the ‘hiring

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<sup>17</sup>See chapter 2 for the details of deriving this standard wage equation.

<sup>18</sup>See online appendix of Ljungqvist and Sargent (2015) for the details of this derivation.

surplus',  $y - w$ , we obtain

$$y - w = y - b - \phi(y - b + \theta c) \quad (4.2.1)$$

Rearranging the equation, we have the following relation,

$$y - w + \phi\theta c = (1 - \phi)(y - b) \quad (4.2.2)$$

Equation (4.2.2) depicts the relation between the 'fundamental surplus' and the 'hiring surplus'. Since the 'hiring surplus' is positively correlated with labour market tightness (see 3.1.3), we conclude that a small (large) 'hiring surplus' leads to a small (large) 'fundamental surplus'.

A major difference between our argument and the 'fundamental surplus' is that in general we regard the sticky wage as a distinct channel and a necessary condition for generating unemployment fluctuations. Whereas Ljungqvist and Sargent (2015) don't specify the role of the sticky wage in generating unemployment fluctuations. To highlight this difference, we assume an ad hoc wage equation,

$$w = \varphi y + \varsigma \theta + \epsilon \quad (4.2.3)$$

where  $\varphi$  and  $\varsigma$  are the coefficients and  $\epsilon$  is constant. We further assume that the wage equation satisfies the following condition,

$$w < y \quad (4.2.4)$$

Condition (4.2.4) says the wage should be smaller than the output. This ensures that the wage satisfies the private efficient condition since it generates a positive surplus to both the firm and the worker. Substituting this wage equation into the job creation condition, we have the following equilibrium condition for labour market tightness,

$$(1 - \varphi)y - \varsigma\theta - \frac{(r + \tau)c}{q(\theta)} - \epsilon = 0 \quad (4.2.5)$$

After implicit differentiation of (4.2.5), we derive the elasticity of labour market tightness with respect to labour productivity

$$\varepsilon_{\theta,y} = \frac{1-\varphi}{\varphi} \frac{y}{y - \frac{w}{\varphi} + \frac{s\theta}{\varphi}} \quad (4.2.6)$$

Under our ad hoc wage specification, the ‘fundamental surplus’ is simply equal to  $y - \frac{w}{\varphi} + \frac{s\theta}{\varphi}$ . If we assume that the wage is fully flexible in the sense that the wage moves proportionally with the productivity, then we have  $\varphi = 1$ . In this case, the elasticity of labour market tightness is 0. This confirms our argument that the sticky wage is a necessary condition for generating unemployment fluctuations. If we substitute the wage equation into the expression for the ‘fundamental surplus’, it turns out the ‘fundamental surplus’ is equal to  $-\epsilon$ . The condition (4.2.4) implies that  $\epsilon$  must be negative when  $\varphi$  is equal to 1. So the ‘fundamental surplus’ is positive. It decreases to 0 when the wage equals output.

This example shows that if the wage were fully flexible, there would be no fluctuations in job vacancies and unemployment even if the ‘fundamental surplus’ is small.

## 5 Large Vacancy Costs

A common feature of the calibration in the post-Shimer literature is giving a large value to the cost of posting a vacancy. This feature is obscured by the fact that the literatures choose different time periods for the calibration. For example, Shimer (2005) assumes that the vacancy cost is 0.213 on a quarterly basis. Whereas Hall and Milgrom (2008) assume that the vacancy cost is 0.43 on a daily basis. A short time period used in calibration rationalises the large value of the vacancy cost. While the diversity of time period in the literature, since the output is always normalized to one, the value of the vacancy cost is independent of the time period. To clarify this issue, we recall the job creation condition

$$y = w + \frac{(r + \tau)}{q(\theta)} c \quad (5.1)$$



In job creation condition, the value of the interest rate, the job separation rate and the vacancy-filling rate all depends on the time period of the model. For instance, if the monthly job separation rate is 0.033, then the quarterly job separation rate should be about 0.1. However, since those three variables appear both on the numerator and denominator of the hiring cost, the ratio  $(r + \tau)/q(\theta)$  is independent of the time period. Therefore the proportion of the wage and the vacancy cost to output is independent of the time period. Since output is normalized to one in all time periods, it turns out that the calibrated value of vacancy cost is also independent of the time period.

For the sake of comparing different calibrations in the literature, we report the values of vacancy costs and equilibrium unemployment used in different models in Table 1<sup>19</sup>. We also transform the values of job separation rates, vacancy-filling rates, labour market tightness, job finding rates and job vacancies across different models into monthly equivalents. We can see that the vacancy cost in the post-Shimer literature at least double that in Shimer (2005).

Table 1—Comparison of Calibrated Parameter Values in the Literature

Parameter	Interpretation	Calibrated Value			
		Shimer (2005)	Hagedorn and Manovskii (2008)	Hall and Milgrom (2008)	Hall (2005)
$c$	Vacancy Cost	0.213	0.584	0.433	0.986
$\tau$	Separation Rate	0.033	0.032	0.030	0.034
$\theta$	Market Tightness	1.000	0.634	0.500	0.539
$q(\theta)$	Vacancy Filling Rate	0.450	0.877	0.960	1.127
$f(\theta)$	Job Finding Rate	0.450	0.556	0.480	0.607
$v$	Job Vacancy	0.068	0.035	0.028	0.025
$u$	Unemployment	0.068	0.055	0.055	0.047

Another common feature of the post-Shimer literature is a high vacancy-

<sup>19</sup>We infer the equilibrium unemployment rate in Shimer (2005) based on the job separation rate and job finding rate. Shimer (2005) didn't report the equilibrium unemployment rate.

filling rate<sup>20</sup>. We already know that the small ‘hiring surplus’ contributes to this. Now we turn to investigate whether the large vacancy cost also makes a contribution to the high vacancy-filling rate. To do so, we establish the following proposition.

**Proposition 4** *If the wage elasticity with respect to the vacancy-filling rate satisfies the following condition  $\varepsilon_{w,q(\theta)} < \frac{\lambda}{w}$ , then we have  $\partial q(\theta)/\partial c > 0$ .*

**Proof.** Rewrite the job creation condition as

$$q(\theta) = \frac{(r + \tau)c}{y - w}. \quad (5.2)$$

Taking the derivative of the vacancy-filling rate to the cost of posting a vacancy,

$$\frac{\partial q(\theta)}{\partial c} = \frac{(r + \tau)(y - w) + (r + \tau)c \frac{\partial w}{\partial q(\theta)} \frac{\partial q(\theta)}{\partial c}}{(y - w)^2} \quad (5.3)$$

Rearranging equation (5.3),

$$[(y - w)^2 - (r + \tau)c \frac{\partial w}{\partial q(\theta)}] \frac{\partial q(\theta)}{\partial c} = (r + \tau)(y - w) > 0. \quad (5.4)$$

To ensure  $\partial q(\theta)/\partial c > 0$ , the following condition should be satisfied,

$$(y - w)^2 - (r + \tau)c \frac{\partial w}{\partial q(\theta)} > 0. \quad (5.5)$$

Substituting equation (5.2) into (5.5),

$$y - w > q(\theta) \frac{\partial w}{\partial q(\theta)}. \quad (5.6)$$

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<sup>20</sup> A common target of calibration is to match the average post-war U.S unemployment rate and job finding rate. Therefore the calibrated values of those two variables in the literature is pretty close to observed values.

Then multiplying both sides of (5.6) with  $w$  and replace  $y - w$  by  $\lambda$ , condition (5.6) can be written as

$$\frac{\lambda}{w} > \varepsilon_{w,q}(\theta) \quad (5.7)$$

■

Proposition 4 shows that the vacancy-filling rate increases with the cost of posting a vacancy if a condition on the wage elasticity is satisfied. The wage elasticity with respect to the vacancy-filling rate is negative under both Nash bargaining and strategic bargaining. In both cases an increase in vacancy-filling rate will increase the firm's threat point in wage bargaining, therefore decreasing the wage. The wage elasticity is simply equal to zero when the wage is fixed as in Hall (2005). Therefore the wage condition in proposition 4 is satisfied in these papers. So the vacancy-filling rate does increase with the vacancy cost in the literature.

According to Proposition 1 and Corollary 3, we argue that unemployment fluctuations also increase with the vacancy cost. So the large calibrated value of posting a job vacancy makes a contribution to generating the large unemployment volatility.

## 6 Matching Efficiency

From Table 1, we can see that despite the large difference in equilibrium job vacancies, the calibrated unemployment rates in each model are similar. This indicates that each model assumes different matching efficiencies. To compare the matching efficiency, we carry out the following exercise. We assume that the job separation rate is 3% per month and that the equilibrium unemployment rate is 5% per month in all the models. Then we calculate how many job vacancies are needed, based on the matching function in the different models, to obtain an unemployment rate of 5%. Our results are reported in Table-2. Not surprisingly, the model with higher matching efficiency requires fewer job vacancies.

Table 2—Comparisons with Matching Efficiency

Model	Matching Function	Parameters		
		$\tau$	$u$	$v$
Shimer (2005)	$0.452u^{0.72}v^{0.28}$	0.03	0.05	0.115
Hagedorn and Manovskii (2008)	$\frac{uv}{(u^{0.407}+v^{0.407})^{\frac{1}{0.407}}}$	0.03	0.05	0.055
Hall and Milgrom (2008)	$0.68u^{0.5}v^{0.5}$	0.03	0.05	0.035
Hall (2005)	$0.947u^{0.235}v^{0.765}$	0.03	0.05	0.026

A high matching efficiency is a necessary calibration choice to offset the effect of low job vacancies on the equilibrium unemployment rate. Next we study how matching efficiency affects the vacancy-filling rate and unemployment volatility. Without loss of generality, we assume the matching function takes the form of

$$M = mM(u, v) \quad (6.1)$$

where  $m$  is the matching coefficient. Following the convention, we assume the matching process is constant return to scale. Therefore, the vacancy-filling rate can be written as

$$q(\theta) = \frac{mM(u, v)}{v} = mM\left(\frac{1}{\theta}, 1\right) \quad (6.2)$$

To proceed, we define  $\alpha$  as the elasticity of matching with respect to unemployment,  $\alpha = -q'(\theta)\theta/q(\theta)$ . Another way to interpret  $\alpha$  is that it measures the weight on job seekers in matching process. A larger  $\alpha$  means the matching process is more reliant on job seekers. We assume  $0 < \alpha < 1$ .

## 6.1 The Matching Coefficient $m$

To investigate how parameter  $m$  influences the vacancy-filling rate, we establish the following proposition.

**Proposition 5** For any  $m > 0$ , we have  $\frac{\partial q(\theta)}{\partial m} < 0$  if  $-\frac{1}{\alpha} \frac{y-w}{w} < \varepsilon_{w,\theta} < 0$ ;  $\frac{\partial q(\theta)}{\partial m} > 0$  if  $\varepsilon_{w,\theta} > 0$  or  $\varepsilon_{w,\theta} < -\frac{1}{\alpha} \frac{y-w}{w}$ ;  $\frac{\partial q(\theta)}{\partial m} = 0$  if  $\varepsilon_{w,\theta} = 0$ .

**Proof.** Take the partial derivative of (6.2) with respect to the matching coefficient  $m$ ,

$$\frac{\partial q(\theta)}{\partial m} = M\left(\frac{1}{\theta}, 1\right) + m \frac{\partial M\left(\frac{1}{\theta}, 1\right)}{\partial \theta} \frac{\partial \theta}{\partial m} \quad (6.3)$$

Substituting (6.2) into the job creation condition (2.1.10),

$$y = w(\theta) + \frac{(r + \tau)c}{mM\left(\frac{1}{\theta}, 1\right)} \quad (6.4)$$

Rearranging equation (6.4),

$$[y - w(\theta)]mM\left(\frac{1}{\theta}, 1\right) = (r + \tau)c \quad (6.5)$$

Taking the derivative of labour market tightness with respect to the matching coefficient  $m$ ,

$$(y - w)M\left(\frac{1}{\theta}, 1\right) + (y - w)m \frac{\partial M\left(\frac{1}{\theta}, 1\right)}{\partial \theta} \frac{\partial \theta}{\partial m} - mM\left(\frac{1}{\theta}, 1\right) \frac{\partial w}{\partial \theta} \frac{\partial \theta}{\partial m} = 0 \quad (6.6)$$

Writing the equation (6.6) as a function of  $\frac{\partial \theta}{\partial m}$ ,

$$\frac{\partial \theta}{\partial m} = \frac{(y - w)M\left(\frac{1}{\theta}, 1\right)}{mM\left(\frac{1}{\theta}, 1\right) \frac{\partial w}{\partial \theta} - (y - w)m \frac{\partial M\left(\frac{1}{\theta}, 1\right)}{\partial \theta}} \quad (6.7)$$

Substituting (6.7) into (6.3) to replace  $\frac{\partial \theta}{\partial m}$ ,

$$\frac{\partial q(\theta)}{\partial m} = M\left(\frac{1}{\theta}, 1\right) + \frac{(y - w)m \frac{\partial M\left(\frac{1}{\theta}, 1\right)}{\partial \theta} M\left(\frac{1}{\theta}, 1\right)}{mM\left(\frac{1}{\theta}, 1\right) \frac{\partial w}{\partial \theta} - (y - w)m \frac{\partial M\left(\frac{1}{\theta}, 1\right)}{\partial \theta}} \quad (6.8)$$

Combining the two terms on the right hand side of (6.8),

$$\frac{\partial q(\theta)}{\partial m} = \frac{M(\frac{1}{\theta}, 1) \frac{\partial w}{\partial \theta}}{M(\frac{1}{\theta}, 1) \frac{\partial w}{\partial \theta} - (y - w) \frac{\partial M(\frac{1}{\theta}, 1)}{\partial \theta}} M(\frac{1}{\theta}, 1) \quad (6.9)$$

Multiplying both the numerator and denominator of the right hand side of (6.9) by  $\theta/M(\frac{1}{\theta}, 1)$ ,

$$\frac{\partial q(\theta)}{\partial m} = \frac{\theta \frac{\partial w}{\partial \theta}}{\theta \frac{\partial w}{\partial \theta} - (y - w) \frac{\partial M(\frac{1}{\theta}, 1)}{\partial \theta} \frac{\theta}{M(\frac{1}{\theta}, 1)}} M(\frac{1}{\theta}, 1) \quad (6.10)$$

By definition  $\frac{\partial M(\frac{1}{\theta}, 1)}{\partial \theta} \frac{\theta}{M(\frac{1}{\theta}, 1)} = -\frac{1}{\alpha}$ , substitute this into (6.10),

$$\frac{\partial q(\theta)}{\partial m} = \frac{\theta \frac{\partial w}{\partial \theta}}{\theta \frac{\partial w}{\partial \theta} + (y - w) \frac{1}{\alpha}} M(\frac{1}{\theta}, 1) \quad (6.11)$$

Then dividing the numerator and denominator by  $w$ ,

$$\frac{\partial q(\theta)}{\partial m} = \frac{\varepsilon_{w,\theta}}{\varepsilon_{w,\theta} + \frac{(y-w)}{w} \frac{1}{\alpha}} M(\frac{1}{\theta}, 1) \quad (6.12)$$

In equation (6.12), both  $M(\frac{1}{\theta}, 1)$  and  $\frac{(y-w)}{w} \frac{1}{\alpha}$  are definitely positive. Now consider three cases for  $\varepsilon_{w,\theta}$ ,

1. If  $\varepsilon_{w,\theta} = 0$ , which is the case when wage is independent with  $\theta$ , then we have

$$\frac{\partial q(\theta)}{\partial m} = 0 \quad (6.13)$$

2. If  $\varepsilon_{w,\theta} > 0$ , which is the case when wage is determined by the Nash bargaining or the strategic bargaining, then we have

$$\frac{\partial q(\theta)}{\partial m} > 0 \quad (6.14)$$

3. If  $\varepsilon_{w,\theta} < 0$ , then the condition for  $\frac{\partial q(\theta)}{\partial m} > 0$  is

$$\varepsilon_{w,\theta} < -\frac{1}{\alpha} \frac{y - w}{w} \quad (6.15)$$

If  $-\frac{1}{\alpha} \frac{y-w}{w} < \varepsilon_{w,\theta} < 0$ , then  $\frac{\partial q(\theta)}{\partial m} < 0$  holds. ■

Proposition 5 shows that the impact of the matching coefficient on the vacancy-filling rate depends on how the wage responds to labour market tightness. Under Nash bargaining and strategic bargaining, the vacancy-filling rate increases with the matching coefficient. According to Proposition 1 and Corollary 3, the unemployment fluctuations also increases with the matching coefficient. However, this conclusion does not apply to the fixed wage case as in Hall (2005).

To investigate the causal chain behind proposition 5, we suppose there is an increase in matching efficiency. This shortens the duration of filling a job vacancy and therefore encourages firms to create more job vacancies. This leads labour market tightness initially to increase.

If the wage had no response to the increase in labour market tightness, then the job creation condition implies that the equilibrium vacancy-filling rate would be the same as before the increase in matching efficiency. So firms would continue to create new job vacancies until the vacancy-filling rate has decreased to the previous equilibrium level.

If the wage were positively correlated with labour market tightness, like it is under Nash bargaining, then the increase in labour market tightness would drive up the wage, diminishing the firm's 'hiring surplus'. According to the job creation condition, a smaller hiring cost is required to achieve a new equilibrium. This requires a higher equilibrium vacancy-filling rate.

If the wage were negatively correlated with labour market tightness, then the increase in labour market tightness would drive down the wage, therefore expanding the firm's 'hiring surplus'. According to the job creation condition, this requires a larger hiring cost to achieve a new equilibrium. This requires a lower vacancy-filling rate in equilibrium. However if the drop of the wage were too dramatic so that workers feel better being unemployed rather than employed and choose to stay in the unemployment pool, then the wage wouldn't decrease and labour market tightness wouldn't change. The vacancy-filling rate would increase simply due to the increase in matching efficiency.

## 6.2 The Matching Elasticity $\alpha$

To study the matching elasticity, we adopt the conventional Cobb-Douglas matching function used in section 2. The vacancy-filling rate can then be written as

$$q(\theta) = m\theta^{-\alpha} \quad (6.16)$$

By taking the derivative of the vacancy-filling rate with respect to the matching elasticity  $\alpha$ , we establish the following proposition:

**Proposition 6** *For any  $0 < \alpha < 1$ , when  $\varepsilon_{w,\theta} > -\frac{1}{\alpha} \frac{y-w}{w}$ , we have  $\frac{\partial q(\theta)}{\partial \alpha} < 0$  if  $\theta \varepsilon_{w,\theta} > 0$ ;  $\frac{\partial q(\theta)}{\partial \alpha} = 0$  if  $\theta = 1$ ; and  $\frac{\partial q(\theta)}{\partial \alpha} > 0$  if  $\theta \varepsilon_{w,\theta} < 0$ .*

**Proof.** Taking the partial derivative of (6.16) with respect to the matching elasticity,

$$\frac{\partial q(\theta)}{\partial \alpha} = -m\theta^{-\alpha} \left( \ln \theta + \frac{\alpha}{\theta} \frac{\partial \theta}{\partial \alpha} \right) \quad (6.17)$$

To derive an equilibrium condition for labour market tightness, we substitute (6.16) into the job creation condition (2.1.10),

$$(y - w)m\theta^{-\alpha} = (r + \tau)c \quad (6.18)$$

Taking the derivative of labour market tightness with respect to the matching efficiency,

$$-(y - w)m\theta^{-\alpha} \left( \ln \theta + \frac{\alpha}{\theta} \frac{\partial \theta}{\partial \alpha} \right) - m\theta^{-\alpha} \frac{\partial w}{\partial \theta} \frac{\partial \theta}{\partial \alpha} = 0 \quad (6.19)$$

Rearrange equation (6.19),

$$\left[ \alpha(y - w)m\theta^{-\alpha-1} + m\theta^{-\alpha} \frac{\partial w}{\partial \theta} \right] \frac{\partial \theta}{\partial \alpha} = -(y - w)m\theta^{-\alpha} \ln \theta \quad (6.20)$$

Solving for  $\frac{\partial \theta}{\partial \alpha}$ ,



$$\frac{\partial \theta}{\partial \alpha} = \frac{-(y-w)m\theta^{-\alpha} \ln \theta}{\alpha(y-w)m\theta^{-\alpha-1} + m\theta^{-\alpha} \frac{\partial w}{\partial \theta}} \quad (6.21)$$

Substituting (6.21) into (6.17) to replace  $\frac{\partial \theta}{\partial \alpha}$ ,

$$\frac{\partial q(\theta)}{\partial \alpha} = -m\theta^{-\alpha}(\ln \theta + \frac{\alpha}{\theta} \frac{-(y-w)m\theta^{-\alpha} \ln \theta}{\alpha(y-w)m\theta^{-\alpha-1} + m\theta^{-\alpha} \frac{\partial w}{\partial \theta}}) \quad (6.22)$$

Rearrange equation (6.22),

$$\frac{\partial q(\theta)}{\partial \alpha} = -m\theta^{-\alpha} \frac{m\theta^{1-\alpha} \ln \theta \frac{\partial w}{\partial \theta}}{\alpha(y-w)m\theta^{-\alpha} + m\theta^{1-\alpha} \frac{\partial w}{\partial \theta}} \quad (6.23)$$

To see whether  $\frac{\partial q(\theta)}{\partial \alpha}$  is a negative number, we consider two cases:

I. If  $\varepsilon_{w,\theta} > -\frac{1}{\alpha} \frac{y-w}{w}$ , we have  $\frac{\partial q(\theta)}{\partial \alpha} < 0$  if  $\left\{ \begin{array}{l} \theta > 1, \varepsilon_{w,\theta} > 0 \\ \theta < 1, \varepsilon_{w,\theta} < 0 \end{array} \right\}$ ;

$$\frac{\partial q(\theta)}{\partial \alpha} > 0 \text{ if } \left\{ \begin{array}{l} \theta > 1, \varepsilon_{w,\theta} < 0 \\ \theta < 1, \varepsilon_{w,\theta} > 0 \end{array} \right\};$$

$$\frac{\partial q(\theta)}{\partial \alpha} = 0 \text{ if } \theta = 1.$$

II. If  $\varepsilon_{w,\theta} < -\frac{1}{\alpha} \frac{y-w}{w}$ , we have  $\frac{\partial q(\theta)}{\partial \alpha} < 0$  if  $\theta > 1$ ;

$$\frac{\partial q(\theta)}{\partial \alpha} > 0 \text{ if } \theta < 1;$$

$$\frac{\partial q(\theta)}{\partial \alpha} = 0 \text{ if } \theta = 1. \blacksquare$$

Proposition 6 establishes the conditions under which the vacancy-filling rate increases with  $\alpha$ . In a slack labour market meaning that the number of job seekers is larger than the number of job vacancies, increasing  $\alpha$  would initially increase the vacancy-filling rate. This is because the matching process more relies on its major participants the job seekers. A higher vacancy-filling rate encourages job creation, driving up labour market tightness. The new labour market equilibrium depends on how the wage reacts

to labour market tightness. If the wage does not respond to the increase in labour market tightness, then the firm's 'hiring surplus' will not change. The job creation condition implies that firms will continue to create new job vacancies until the vacancy-filling rate decreases to the previous equilibrium level. If the wage increases with labour market tightness, as under Nash or strategic bargaining, then firm's 'hiring surplus' will decrease. The job creation condition implies that a higher vacancy-filling rate is required to achieve a new equilibrium. If the wage decreases with labour market tightness, then firm's 'hiring surplus' will increase. The job creation condition implies that a lower vacancy-filling rate is required to achieve a new equilibrium.

In a tight labour market meaning that the number of job vacancies is larger than the number of job seekers, increasing  $\alpha$  would initially decrease the vacancy-filling rate. This is because the matching process now more relies on its minor participants the job seekers. A lower vacancy-filling rate discourages the job creation, so labour market tightness decreases. A new labour market equilibrium still depends on how the wage reacts to the labour market tightness, as discussed above.

If the labour market has the same amount of job seekers and job vacancies, it makes no difference which side has a larger impact on the matching process. Therefore increasing  $\alpha$  has no effect on the vacancy-filling rate.

## 7 Conclusion

In this chapter, we argue that a small 'hiring surplus' is a solution to the unemployment volatility puzzle. This solution has been discussed in the literature and we are not the first to consider it, see Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2015). The novelty in this chapter is that we are the first to lay out the mechanism behind this solution. We point out that a high vacancy-filling rate, as a result of the small 'hiring surplus', plays the key role in generating unemployment volatility. A higher vacancy-filling rate ensures that any variation of job vacancies has a larger impact on the variation of unemployment.

We also develop the argument on sticky wages as a solution to the unemployment volatility puzzle. We argue that the sticky wage ensures that the job vacancies vary with the labour productivity. This is essential because when the job separation is assumed to be constant, the variation of job vacancies is the only source for the variation of unemployment.

In the second half of the chapter, we argue that a common calibration strategy in the literature also makes contribution to generating large unemployment volatility. This calibration strategy raises the relative cost of creating a job vacancy and increases the matching efficiency. A large cost of creating a job vacancy may reflect some other frictions firms have to face. For example, firms need to obtain the loans from financiers before setting up new positions, which will increase the vacancy cost. On the other hand, a high matching efficiency implies small search and matching frictions in the labour market.

## Chapter 4

# Strategic Bargaining, the Discount Rate and Endogenous Job Destruction

### Abstract

This chapter contains a critical review of recent developments in the literature on the unemployment volatility puzzle. Specifically, we reassess the arguments on: strategic wage bargaining; large fluctuations in discount rates in the financial market; and endogenous job separations caused by idiosyncratic productivity shocks.

# 1 Introduction

Based on bargaining theory by Binmore, Rubinstein and Wolinsky (see Rubinstein 1982, Binmore, Rubinstein and Wolinsky 1986), the strategic wage bargaining, as argued by Hall and Milgrom (2008), challenges one unrealistic assumption of Nash wage bargaining: the threat made by counterparties during a wage bargain to walk away and terminate the bargain. This is unrealistic in the sense that bargainers wouldn't so easily abandon the joint surplus arising from search frictions. Given this fact, the threat under wage bargaining is better expressed as to extend bargaining rather than to terminate it.

This changes the determinants of the wage in the DMP model. Under the threat of terminating the bargain in Nash bargaining, the wage takes into account the value of outside options for the worker. This largely depends on the probability that the worker can find a new employer and the expected wage payment. So it is endogenous to the model and responsive to the productivity shock. Under the threat of delay in the bargain in strategic bargaining, the wage takes into account the cost of delay in wage negotiation<sup>21</sup>. This cost is assumed to be exogenous and constant. One implication of this change is that the wage will be less responsive to the productivity shock, which leads a firm's surplus and recruiting effort to be more variable in response to the productivity shock.

Although the fixed cost of delay is a conventional assumption in bargaining theory, it is not a realistic assumption in the search frictions model. The literature does not specify what is the cost of delay in wage negotiation. We think there can be two basic components. One is the 'menu cost', which is the cost spent on initiating a bargaining round. The other is the opportunity cost, which is the lost production due to the delay of reaching a wage agreement. It might be true that the 'menu cost' is fixed but the opportunity cost is definitely pro-cyclical. Since the opportunity cost is more likely

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<sup>21</sup>Under strategic bargaining, the bargaining can be terminated for some reasons and workers will go back to unemployment pool. However this only has a secondary influence on the wage.

to be the main component, it is more realistic to assume a pro-cyclical cost of delay in wage negotiation.

In the literature, the cost of delay accounts for a large proportion of productivity (see Hall and Milgrom (2008) and Hall (2015)). So it has a significant influence on wage flexibility. In this chapter, we argue that if we adopt a more realistic assumption on the cost of delay, the model with strategic wage bargaining cannot replicate the observed unemployment fluctuations under plausible calibrations of structural parameters. However it is still better than the model with Nash bargaining.

There is a rapidly growing literature focusing on the observed co-movement between job hiring and discount rates in the U.S stock market. One of the most recent papers, written by Hall (2015), suggests that the discount rate in the stock market is a driving force of unemployment fluctuations in the U.S economy. His argument heavily relies on two hypotheses. One is that the future stream of benefits from a new hire is discounted using the discount rate in the stock market. During the financial crisis, the risk premium rises. So the discount rate in the stock market rises. This leads to a lower expected value of a new job to a firm. Therefore firms have a lower incentive to post job vacancies.

The second hypothesis is that the wage is insulated from fluctuations in the labour and stock markets. This is essential in Hall (2015) because if the wage decreases as its response to the discount rate or labour market tightness, then firms' incentives for job creation will be restored. If so, the impact of discount rates fluctuations on unemployment volatility is rather limited. To avoid such effects, Hall (2015) carefully specifies the wage equation so that the wage is totally independent of the discount rate and labour market tightness.

Given the co-movement between U.S labour market tightness and the S&P stock market index, we agree with the basic conclusion reached by Hall (2015) that large fluctuations of discount rates in stock market can be a driving force of unemployment volatility. However we have doubts about the mechanism Hall (2015) proposed. Specifically, we doubt that the discount rate is the main channel that transmits fluctuations in financial

markets to job hiring. We show that if Hall’s hypothesis were true, then if we endogenize job destruction in the standard DMP model, one implication would be that job destructions decrease with the discount rates in the stock market, which contradicts the observed facts in financial crisis.

We also show that Hall (2015)’s results rely heavily on his wage specification. If the wage regime is switched to standard Nash bargaining, the impact of discount rates on labour market volatility becomes trivially small. To our best knowledge, the literature is far from reaching a consensus on wage flexibility. In spite of this, requiring a specific wage specification indicates that the discount rate channel in Hall (2015) is probably over-emphasized.

Most of the literature on unemployment volatility assumes a constant exogenous rate of job separation. Therefore modelling the volatility of job vacancy and unemployment entirely reflects fluctuations in job creation. However U.S data shows that both job creation and job destruction respond to exogenous shocks (see Davis, Haliwanger and Schuh, 1996). As one of few responses to this fact, Fujita and Ramey (2012) examine the role of job destruction in generating unemployment volatility. They argue that under a modest value of leisure, endogenous job separation can enhance the ability of the DMP model to produce realistic unemployment volatility. However their model cannot fully match the observed data. Due to the complexity of modelling both the job creation and job separation, their model is also far less intuitive.

We show that endogenous job separation can increase the volatility of labour market tightness, especially when reservation productivity is high. Reservation productivity is the lower bound of the production below which job separation occurs. High reservation productivity triggers large flows into unemployment. Large inflows to unemployment slacken the labour market, increasing the outflows from unemployment. The two larger flows lead to a more volatile labour market.

We also show that reservation productivity is negatively correlated with aggregate productivity. One implication is that job destruction may not be a key factor for generating unemployment volatility in ‘normal’ periods, but probably is a key factor in recessions.

The chapter proceeds as follows. Section 2 lays out the basic DMP model. The strategic wage bargaining and the impact of fixed cost of delay in wage negotiation on labour market volatility are discussed in Section 3. In Section 4, the relation between high discount rates in stock market and labour market tightness is discussed. Section 5 investigates the implications of endogenous job destruction, and Section 6 concludes.

## 2 Search and Matching Model

To set the stage, we review key questions and equilibrium relationships for a basic discrete time search and matching model. There is a continuum of identical individuals on the unit interval. Each individual inelastically supplies one unit of labour in every period and consumes all the income they earn. They are infinitely lived and risk neutral with discount factor  $\beta = (1 + r)^{-1}$  where  $r$  is the discount rate. An individual is either employed and earning a wage  $w$ , or else unemployed and enjoying utility of leisure  $b$ . If unemployed, an individual finds a job with probability  $f$ . At the end of each period, existing job matches are terminated with probability  $\tau$ . Since we assume that all firms are identical, the value of being employed is thus

$$L = w + \beta[\tau U + (1 - \tau)L] \quad (2.1)$$

while the value of being unemployed is

$$U = b + \beta[fL + (1 - f)U] \quad (2.2)$$

There is a continuum of identical firms on the unit interval. Each firm can hire up to one worker who produces an amount  $y$ . Existing jobs command rents  $y - w$  for the firms in equilibrium. The value of a filled job is

$$J = y - w + \beta[\tau V + (1 - \tau)J] \quad (2.3)$$



where  $V$  is the value of a vacant job. Firms must pay a real per-period fixed cost of  $c$  at the start of each period to post a vacancy<sup>22</sup>. Vacancies are then filled with probability  $q$ ; if the vacancy is filled, the new job match becomes productive in the following period. The value of a vacant job is then

$$V = -c + \beta[qJ + (1 - q)V] \quad (2.4)$$

Trade in labour market is uncoordinated, time-consuming, and costly for both firms and workers. The number of jobs formed in each period is determined by a matching function  $M(u, v)$  where  $u$  is the number of job seekers and  $v$  are aggregate vacancies. The matching function  $M(u, v)$  is increasing in both arguments, concave and homogeneous of degree one. We proceed under the assumption that the matching function has the Cobb-Douglas form,  $h = mu^\alpha v^{1-\alpha}$  where  $m > 0$  and  $\alpha$  is the constant elasticity of matching with respect to unemployment. Defining  $\theta = \frac{v}{u}$  as a measure of the tightness of the labour market, total hiring  $h$  is

$$h = mu\theta^{1-\alpha} \quad (2.5)$$

The probability of a firm filling a vacancy is

$$q = \frac{h}{v} = m\theta^{-\alpha} \quad (2.6)$$

While the probability that an unemployed worker finds a job is

$$f = \frac{h}{u} = \theta q(\theta) \quad (2.7)$$

The homogeneity of matching function implies  $q(\theta)' < 0, q(\theta)'' > 0$  and  $f(\theta)' > 0, f(\theta)'' < 0$ . It is worth to note  $\alpha = -q'(\theta)\theta/q(\theta)$ .

The number of workers who enter unemployment in each period is  $\tau(1 - u)$ , and the number who leave unemployment is  $\theta q(\theta)u$ . In equilibrium, unemployment is constant, so the two flows are equal,  $\tau(1 - u) = \theta q(\theta)u$ .

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<sup>22</sup>The cyclical cost of posting a vacancy is discussed in the appendix. For simplicity, only a fixed vacancy cost is considered in this section.

We rewrite unemployment rate as the function of two transition rates:

$$u = \frac{\tau}{\tau + \theta q(\theta)} \quad (2.8)$$

Equation (2.8) is the first key equation of the model. It is known as the *Beveridge curve* when represented in vacancy-unemployment space. In equilibrium, free entry in the labour market drives rents from vacant jobs to zero. Therefore the equilibrium condition for vacancy creation is  $V = 0$ , implying that

$$J = \frac{c}{\beta q(\theta)} \quad (2.9)$$

Condition (2.9) states that in equilibrium, the expected profit from a new job is equal to the expected cost of hiring a worker. Substitute equilibrium condition (2.9) into (2.3),

$$y - w - \frac{(r + \tau)c}{q(\theta)} = 0 \quad (2.10)$$

Equation (2.10) describes the marginal condition for the demand for labour. Total output  $y$  is used to pay for the wage  $w$  and the expected capitalized value of the firm's hiring cost  $\frac{(r+\tau)c}{q(\theta)}$ .

### 3 Strategic Wage Bargaining

Hall and Milgrom (2008) address the Shimer critique by replacing Nash axiomatic bargaining with a sequential strategic bargaining approach (see Rubinstein 1982, Binmore, Rubinstein and Wolinsky 1986)<sup>23</sup>. The incentive of the parties to reach agreement depends on the bargainers' time preference and the risk of breakdown of negotiation. The firm and the worker understand they have a strictly higher payoff from reaching agreement rather than breaking up and accepting the outside options. The firm and the worker take

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<sup>23</sup> Christiano, Eichenbaum and Trabandt (2015) incorporate the strategic bargaining approach into a DSGE-type model with search frictions in labour market. They use this approach to study the response of macro variables to monetary shocks.

turns making their wage offer denoted by  $w^f$  and  $w^w$ . In each bargaining round, each party either accepts the counterparty's offer or rejects and proposes a counteroffer. After a delay, the firm incurs a fixed cost of delay  $\gamma$  while the worker enjoys the value of leisure  $b$ . There is a probability  $\delta$  that the job opportunity is exogenously destroyed between bargaining rounds. In that case, the firm and the worker revert to their outside options.<sup>24</sup>

The value of being employed to a worker,  $L$ , and the value of a filled job for a firm,  $J$ , are still given by expressions (2.1) and (2.3). Rearrange these two expressions,

$$L = \frac{w + \beta\tau U}{1 - \beta(1 - \tau)} \quad (3.1)$$

$$J = \frac{y - w}{1 - \beta(1 - \tau)} \quad (3.2)$$

where the free-entry condition,  $V = 0$ , is imposed in (3.2). Suppose the firm makes the first wage offer. The bargaining strategy for the firm is to ensure a worker is indifferent between accepting the firm's wage offer or rejecting and waiting until the next round to make a counteroffer. As a result, the worker will accept the firm's initial wage offer. The optimal condition for firm's wage offer,  $w^f$ , is

$$\frac{w^f + \beta\tau U}{1 - \beta(1 - \tau)} = b + \beta[(1 - \delta)\frac{w^w + \beta\tau U}{1 - \beta(1 - \tau)} + \delta U] \quad (3.3)$$

Rearranging the right hand side of (3.3),

$$\frac{w^f}{1 - \beta(1 - \tau)} = b + \beta(1 - \delta)\frac{w^w}{1 - \beta(1 - \tau)} + \beta\frac{1 - \beta}{1 - \beta(1 - \tau)}(\delta - \tau)U \quad (3.4)$$

The firm would not propose a wage offer lower than  $w^f$  because the worker would reject the firm's wage offer and wait until the next round to make a counteroffer. In that case, the firm has to pay a fixed cost of delay and it is unlikely to receive an offer higher than  $w^f$ . Also, there is no reason for

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<sup>24</sup>For the worker, the outside option is unemployment, which has value  $U$ . For the firm, the outside option is simply to quit the labour market.

the firm to propose a wage offer higher than  $w^f$ . Hall and Milgrom (2008) argue that ‘the limited influence of unemployment on the wage results in large fluctuations in unemployment...’. To highlight this, we assume  $\delta = \tau$  so that  $U$  does not influence  $w^f$ ,

$$w^f = [1 - \beta(1 - \delta)]b + \beta(1 - \delta)w^w \quad (3.5)$$

Similarly the optimal worker’s wage offer will ensure a firm is indifferent between accepting the worker’s initial wage offer and rejecting and making a counteroffer in the next round. The optimal condition for worker’s wage offer,  $w^w$ , is

$$\frac{y - w^w}{1 - \beta(1 - \delta)} = -\gamma + \beta(1 - \delta) \frac{y - w^f}{1 - \beta(1 - \tau)} \quad (3.6)$$

imposing the condition  $\delta = \tau$ ,

$$y - w^w = -[1 - \beta(1 - \delta)]\gamma + \beta(1 - \delta)(y - w^f) \quad (3.7)$$

The expression (3.5) and (3.7) jointly determine  $w^f$  and  $w^w$ . We assume that the firm makes the initial offer. In equilibrium, the first wage offer is accepted. Solving for  $w^f$ ,

$$w = w^f = \frac{b + \beta(1 - \delta)(y + \gamma)}{1 + \beta(1 - \delta)} \quad (3.8)$$

Substitute the wage expression (3.8) into the hiring condition (8) to derive the following expression for equilibrium market tightness

$$\frac{b + \beta(1 - \delta)(y + \gamma)}{1 + \beta(1 - \delta)} = y - \frac{r + \tau}{q(\theta)}c \quad (3.9)$$

After implicit differentiation, we can compute the elasticity of market tightness

$$\eta_{\theta,y} = \frac{1}{\alpha} \frac{y}{y - b - \beta(1 - \delta)\gamma} \quad (3.10)$$

**Proof.** Implicit differentiation of (3.9) yields

$$\frac{d\theta}{dy} = \frac{\frac{1}{1+\beta(1-\delta)}}{-\frac{(r+\tau)cq'(\theta)}{q(\theta)^2}} \quad (3.11)$$

since we know that  $\theta q'(\theta)/q(\theta) = -\alpha$ , substitute this into (3.11)

$$\frac{d\theta}{dy} = \frac{1}{1+\beta(1-\delta)} \frac{1}{\alpha \frac{(r+\tau)\gamma}{\theta q(\theta)}} \quad (3.12)$$

then substitute (3.9) into (3.12) to replace  $\frac{(r+\tau)c}{q(\theta)}$ ,

$$\frac{d\theta}{dy} = \frac{1}{1+\beta(1-\delta)} \frac{\theta}{\alpha \frac{y-b-\beta(1-\delta)\gamma}{1+\beta(1-\delta)}} \quad (3.13)$$

re-arrange the equation, we obtain

$$\frac{d\theta}{dy} = \frac{\theta}{\alpha[y-b-\beta(1-\delta)\gamma]} \quad (3.14)$$

then we derive the elasticity of labour market tightness with respect to productivity as

$$\eta_{\theta,y} = \frac{y}{\alpha[y-b-\beta(1-\delta)\gamma]} \quad (3.15)$$

■

### 3.1 Discussions

Expression (3.10) says that the risk adjusted discounted cost of delay  $\beta(1-\delta)\gamma$  shrinks the firm's resources for vacancy creation as the worker can exploit this cost when bargaining with the firm. This is reflected in wage equation (3.8). Although the fixed cost of delay has been conventionally used in strategic bargaining, it might not be plausible to say that firms simply face a fixed cost of delay. Although there are some material costs of delay for the firm, which probably can be seen as fixed, the opportunity cost of the lost production due to the delay of reaching a wage agreement is arguably

more important. Normally firms have more incentive to hire workers in a economic boom rather than in a recession because firms face a larger opportunity of being idle in a boom. This opportunity cost is procyclical. If we reckon that the opportunity cost is the major cost of delay, it would be more appropriate to assume the cost of delay is procyclical,  $\bar{\gamma}y$ , where  $0 < \bar{\gamma} < 1$ . In that case, expression (3.10) can be written as

$$\frac{b + \beta(1 - \delta)(1 + \bar{\gamma})y}{1 + \beta(1 - \delta)} = y - \frac{r + \tau}{q(\theta)}c \quad (3.16)$$

The elasticity of market tightness now becomes

$$\eta_{\theta,y} = \frac{1 - \beta(1 - \delta)\bar{\gamma}}{\alpha} \frac{y}{y - b - \beta(1 - \delta)\bar{\gamma}} \quad (3.17)$$

**Proof.** Implicit differentiation of (3.16) yields

$$\frac{d\theta}{dy} = \frac{\frac{1 - \beta(1 - \delta)\bar{\gamma}}{1 + \beta(1 - \delta)}}{-\frac{(r + \tau)cq'(\theta)}{q(\theta)^2}} \quad (3.18)$$

since we know that  $\theta q'(\theta)/q(\theta) = -\alpha$ , substitute this into (3.11)

$$\frac{d\theta}{dy} = \frac{1}{1 + \beta(1 - \delta)} \frac{1 - \beta(1 - \delta)\bar{\gamma}}{\alpha \frac{(r + \tau)\bar{\gamma}}{\theta q(\theta)}} \quad (3.19)$$

then substitute (3.9) into (3.12) to replace  $\frac{(r + \tau)c}{q(\theta)}$ ,

$$\frac{d\theta}{dy} = \frac{1}{1 + \beta(1 - \delta)} \frac{\theta[1 - \beta(1 - \delta)\bar{\gamma}]}{\alpha \frac{y - b - \beta(1 - \delta)\bar{\gamma}}{1 + \beta(1 - \delta)}} \quad (3.20)$$

re-arrange the equation, we obtain

$$\frac{d\theta}{dy} = \frac{\theta[1 - \beta(1 - \delta)\bar{\gamma}]}{\alpha[y - b - \beta(1 - \delta)\bar{\gamma}]} \quad (3.21)$$

then we derive the elasticity of labour market tightness with respect to

productivity as

$$\eta_{\theta,y} = \frac{1 - \beta(1 - \delta)\bar{\gamma}}{\alpha} \frac{y}{y - b - \beta(1 - \delta)\bar{\gamma}} \quad (3.22)$$

■

Expression (3.17) shows that  $\beta(1 - \delta)\bar{\gamma}$  appears both in the numerator and denominator of  $\eta$  if the cost of delay is procyclical. Since we have  $y > \alpha$  under a plausible calibration of  $\alpha$ , the role of  $\beta(1 - \delta)\bar{\gamma}$  in propogating  $\eta_{\theta,y}$  is reduced. This is probably because the wage is more cyclical when the cost of delay is cyclical, see (3.8), which leads to the firm's marginal benefit of hiring a worker being less cyclical.

To see the quantitative effect of a procyclical cost of delay on the elasticity of labour market tightness, we calculate the elasticity based on (3.10) and (3.17). Values of calibrated parameters are close to Hall and Milgrom (2008), see Table 1. We normalize a time period to be one quarter. Following Hall and Milgrom (2008), we set the exogenous job separation rate to  $\tau = 0.1$ ; the discount rate is set as  $\beta = 0.988$ ; the cost of delay accounts for 27% of output; matching elasticity is set as  $\alpha = 0.5$  and labour productivity is normalized to one. The value of unemployment compensation is chosen so that when the cost of delay is fixed the elasticity of labour market tightness with respect to productivity matches Shimer (2005)'s estimation based on U.S data. Therefore we set  $b$  equal to 0.66 which is higher than the calibrated value of 0.25 in Hall and Milgrom (2008). This is because to derive (3.10) and (3.17) we set the probability that job opportunity disappears during bargaining  $\delta$  equal to the exogenous job separation rate so the flow value of being unemployed does not enter our wage equation (3.8). To obtain a similar value of equilibrium wage as in Hall and Milgrom (2008), we raise the value of unemployment compensation.

Table 1— Values of Calibrated Parameters		
Parameter	Interpretation	The Value of Calibrated Parameter
$\tau$	Exogenous Separation Rate	0.1
$\beta$	Discount Factor	0.988
$b$	Unemployment Compensation	0.66
$\gamma(\bar{\gamma}y)$	Cost of Delay	0.27
$\alpha$	Matching Elasticity	0.5
$y$	Labour Productivity	1

Table 2 shows the elasticity of labour market tightness under our calibration strategy. When the cost of delay is fixed, the elasticity of labour market tightness is 20 times as large as the elasticity of labour productivity. However if the cost of delay is procyclical to productivity, the volatility of labour market tightness drops sharply. This indicates that the success of Hall and Milgrom (2008) to address unemployment volatility puzzle is largely due to assuming a fixed cost of delay in wage negotiations.

Table 2— Elasticity of Labour Market Tightness		
	$\gamma$ is fixed	$\gamma$ is procyclical
Elasticity of Labour		
Market Tightness	20	15.2

## 4 High Discount Rates

In recent decades the dominant view in the literature is that fluctuations in productivity are the driving force behind labour market volatility. However Hall (2015) points out that unemployment did not seem to track the movements of productivity in the last three recessions in the U.S. He proposes a different view: the observed large fluctuations in financial discounts are a potential driving force of unemployment fluctuations.



The causal chain behind his argument is straightforward. When a financial crisis hits the economy, risk premiums rise, so discount rates rise. The expected discounted value of hiring a worker therefore decreases. So firms put fewer resources into recruiting new workers and unemployment rises.

There is a sharp contrast between Hall (2015) and the existing literature. Hall (2015) assumes that the discounted future stream of contributions from a new hire depends on the discount rate in the stock market. Whereas the convention in literature is to assume the flow of benefits from a newly hired worker is risk-free. The risk-free discount rates decrease rather than increase during financial crisis. One of the implications is that job hiring will be counter-cyclical and unemployment will be pro-cyclical. However job hiring decreases and unemployment increases in a recession. This indicates that under the conventional view the discount rate is not a major driving force of unemployment volatility.

To layout how the discount rate affects labour market tightness in search frictions model, we establish the following proposition.

**Proposition 7** *In a labour market with search and matching frictions with exogenous separation rate  $\tau$ , an individual firm pays  $\frac{c}{q(\theta)}$  to hire a worker. The elasticity of labour market tightness with respect to the discount rate is given by*

$$\varepsilon_{\theta,r} = -\frac{1}{\alpha(y-w)} \left[ \frac{rc}{q(\theta)} + w\varepsilon_{w,r} \right] \quad (4.1)$$

**Proof.** Recall the job creation condition (2.10) and rewrite it as

$$q(\theta)(y-w) = (r+\tau)c \quad (4.2)$$

take the derivative of (4.2) with respect to the discount rate

$$\frac{\partial q(\theta)}{\partial \theta} \frac{\partial \theta}{\partial r} (y-w) - q(\theta) \frac{\partial w}{\partial r} = c \quad (4.3)$$

rearrange the equation (4.3)

$$\frac{\theta}{q(\theta)} \frac{\partial q(\theta)}{\partial \theta} \frac{r}{\theta} \frac{\partial \theta}{\partial r} (y-w) = \frac{rc}{q(\theta)} + \frac{\partial w}{\partial r} r \quad (4.4)$$

since we know that  $\theta q'(\theta)/q(\theta) = -\alpha$ , substitute this into (4.4) and define  $\varepsilon_{\theta,r}$  as  $r\partial\theta/\theta\partial r$  and  $\varepsilon_{w,r}$  as  $r\partial w/w\partial r$ ,

$$-\alpha(y-w)\varepsilon_{\theta,r} = \frac{rc}{q(\theta)} + w\varepsilon_{w,r} \quad (4.5)$$

from (4.5), we obtain

$$\varepsilon_{\theta,r} = -\frac{1}{\alpha(y-w)} \left[ \frac{rc}{q(\theta)} + w\varepsilon_{w,r} \right] \quad (4.6)$$

■

Proposition 7 shows that a higher elasticity of labour market tightness with respect to the discount rate is obtained if firms face (i) a smaller marginal benefit from a newly hired worker; (ii) a larger cost of hiring a new worker; (iii) a larger wage elasticity with respect to the discount rate (if it is positive) or a smaller wage elasticity (if it is negative); (iv) a larger discount rate and (v) a smaller matching elasticity with respect to unemployment.

To determine  $\varepsilon_{w,r}$ , we first consider the wage under Nash wage bargaining. Recall the Nash bargained wage from chapter 3,

$$w = (1-\phi)b + \phi(y + \theta c) \quad (4.7)$$

where  $\phi$  measures the relative bargaining power of workers. So under Nash bargaining, the wage co-moves with labour market tightness. Taking the derivative of (4.7) with respect to the discount rate, we obtain

$$\frac{\partial w}{\partial r} = \phi c \frac{\partial \theta}{\partial r} \quad (4.8)$$

Substituting (4.8) into proposition 7 gives the following corollary

**Corollary 8** *If the wage is determined by Nash bargaining, the elasticity of labour market tightness with respect to the discount rate can be written as*

$$\varepsilon_{\theta,r} = -\frac{1}{\alpha(y-w) + \phi\theta c} \frac{rc}{q(\theta)} \quad (4.9)$$

**Proof.** Substitute (4.8) into (4.4), we obtain

$$\frac{\theta}{q(\theta)} \frac{\partial q(\theta)}{\partial \theta} \frac{r}{\theta} \frac{\partial \theta}{\partial r} (y - w) = \frac{rc}{q(\theta)} + \phi c \frac{\partial \theta}{\partial r} r \quad (4.10)$$

since we know that  $\theta q'(\theta)/q(\theta) = -\alpha$ , substitute this into (4.10) and define  $\varepsilon_{\theta,r}$  as  $r \partial \theta / \theta \partial r$ ,

$$-[\alpha(y - w) + \phi \theta c] \varepsilon_{\theta,r} = \frac{rc}{q(\theta)} \quad (4.11)$$

rearranging (4.11) we obtain (4.9). ■

## 4.1 Discussions

Corollary 8 implies that the elasticity of labour market tightness with respect to the discount rate is small when the wage is determined by Nash bargaining. This is largely due to the comovement between the wage and labour market tightness. Specifically, suppose risk premiums rise in a financial crisis, so interest rates rise. The expected discounted value of hiring a worker therefore decreases. Firms put fewer resources into vacancy creation and labour market tightness decreases. Under Nash bargaining, the wage also decreases, especially when the worker's bargaining power is larger since the wage is more strongly correlated with labour market tightness in this case. The decrease in the wage increases the expected discounted value of hiring a worker. This largely offsets the negative effect of increase in discounts rates on hiring.

From this example we see that how the wage responds to movements in discounts rates is crucial for  $\varepsilon_{\theta,r}$ . Hall (2015) specifies the wage using the strategic bargaining approach. Specifically, Hall (2015) assumes that the actual wage is the average of the wages proposed by the firm and by the worker. To determine the wage equation under strategic bargaining, recall the firm's proposal for the wage, (3.8), which we derived in the previous section,

$$w^f = \frac{b + \beta(1 - \delta)(y + \gamma)}{1 + \beta(1 - \delta)} \quad (4.12)$$

Substituting this into (3.5), we can infer the worker's proposed wage,

$$w^w = \frac{y + \gamma + \beta(1 - \delta)b}{1 + \beta(1 - \delta)} \quad (4.13)$$

The actual wage is the average of the two proposed wages, so we have

$$w = \frac{w^f + w^w}{2} = \frac{1}{2}(y + b + \gamma) \quad (4.14)$$

An important feature of wage equation (4.14) is that the wage neither responds to the movements in discounts rates nor responds to the movements in labour market tightness. This result is robust to a more general specification when  $\delta \neq \tau$ . Under Hall's wage specification, we can conclude that

$$\varepsilon_{w,r} = 0 \quad (4.15)$$

Substituting this into (4.6), we obtain

$$\varepsilon_{\theta,r} = -\frac{1}{\alpha(y - w)} \frac{rc}{q(\theta)} \quad (4.16)$$

Comparing (4.16) with (4.9) in corollary 2, we can see that if the two wage regimes imply a similar level of wage and labour market tightness, then labour market tightness is more volatile under strategic bargaining. To quantitatively measure the effect of the two wage regimes on  $\varepsilon_{\theta,r}$ , we calibrate the models as follows. Following Hall (2015), we normalize a time period to be one month. The calibration of the model with strategic bargaining is exactly the same as Hall (2015). In the model with Nash bargaining, the value of worker's bargaining power and unemployment compensation are chosen so that the model can match the vacancy filling rate in Hall (2015). We use the same calibration strategy as Hall (2015) to calibrate the rest of the structural parameters in the model with Nash bargaining. Table 3 displays the calibration values of structural parameters. The last three rows report the steady-state values of endogenous variables.

Table 3—Calibration with Nash and Strategic Bargaining

Parameter	Interpretation	The Value	
		N.B.	S.B.
$\phi$	Worker's Bargaining Power	0.6	—
$\delta$	Interruption Rate	—	0.013
$\gamma$	Fixed Cost of Delay	—	0.567
$\tau$	Exogenous Job Separation	0.0345	0.0345
$r$	Interest Rate	0.01	0.01
$b$	Unemployment Compensation	0.6	0.4
$c$	Vacancy Cost	0.213	0.213
$\alpha$	Matching Elasticity	0.5	0.5
$y$	Labour Productivity	1	1
$\omega$	Wage	0.989	0.984
$\theta$	Labour Market Tightness	1.1	0.59
$q(\theta)$	Vacancy Filling Rate	0.924	1

Table 4 reports the elasticity of labour market tightness with respect to the discount rate under two wage regimes. Not surprisingly, labour market tightness is much more volatile when wage is determined by strategic bargaining.

Table 4— Elasticity of Labour Market Tightness

	Nash Wage	Strategic Wage
	Bargaining	Bargaining
Elasticity of Labour		
Market Tightness	-0.016	-0.266
$\varepsilon_{\theta,r}^{S.B} / \varepsilon_{\theta,r}^{N.B}$		16.63

Overall the findings in Hall (2015) are interesting but not definitive. His conclusion that large fluctuations in discount rates in financial market are a driving force of the unemployment volatility relies on two assumptions. One is that firms evaluate the value of a job based on the discount rate in the stock market. The other is that the wage is insulated from the discount rate and labour market tightness.

Hall (2015) argues that the strong correlation between the value of a job and the S&P stock-market index confirms the first assumption, see pp.20, figure-6. To infer the value of a job to the firm,  $J$ , he constructs a time series of the vacancy-filling rate,  $q$ , based on Job Openings and Labour Turnover Survey (JOLTS) data on hiring rate and the number of vacancies, from 2001 through 2014. Then he uses this to calculate the cost of hiring a worker,  $c/q$ ,<sup>25</sup>. The free entry condition in the labour market implies  $J = c/q$ . Given the way he measures the value of a job, it might be better to say the figure-6 in pp.20 indicates a strong correlation between labour market tightness and the S&P stock-market index. In figure 1, we construct a time series of labour market tightness based on JOLTS monthly reports on number of vacancies and unemployment, from 2001 through 2015, and we compare it with the S&P stock-market index. The similarity between the S&P index and labour market tightness confirms our argument.

Although U.S stock market data show a positive correlation with the value of a job estimated by Hall (2015), we can't infer that the discount rate is the major driving force behind this correlation. If we rewrite the value function for a job, (2.3), as

$$J = \frac{y - w}{1 - \beta(1 - \tau)} \quad (4.17)$$

we can see that the job value is positively correlated with the firm's net profit,  $y - w$ , and the discount factor  $\beta$ , and negatively correlated with job separation rate  $\tau$ . The firm's net profit and the discount factor are both positively correlated with stock market value. The job separation rate is negatively correlated with stock market value at least in recent two recessions<sup>26</sup>. Therefore it is hard to say which factor is the major driving force.

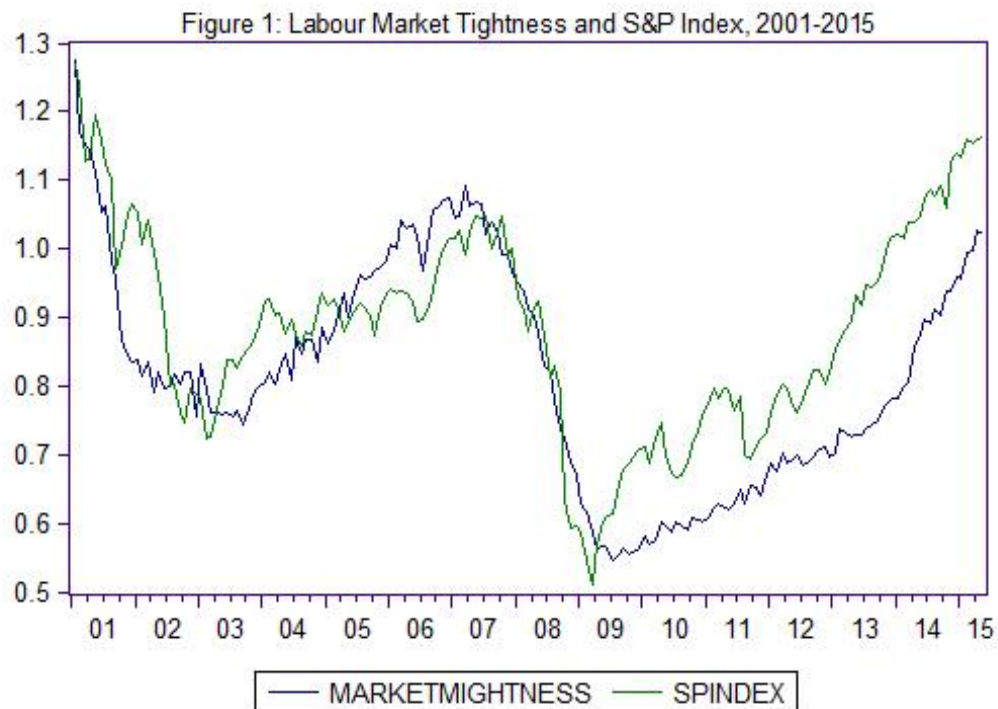
In the next section with endogenous job destruction, we point out another issue behind the first assumption.

Even though we doubt some assumptions in Hall (2015), we do acknowledge the fact that the interest rates can link the dynamics of labour and

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<sup>25</sup>In a discrete time DMP model, the cost of hiring a worker is  $\frac{c}{\beta q}$ .

<sup>26</sup>The time series of layoffs is displayed in figure 2. Comparing with figure 1 and 2, we can see a negative correlation between job separation and stock market value, especially in the recent two recessions.



financial markets, given the large evidence in the literature (see Hall (2016) for a detailed survey). Studying this linkage is an interesting topic for future research.

## 5 Endogenous Job Destruction

Empirical studies suggest that job destruction might be an important source of unemployment volatility. In this section, we give an analysis of the impact of endogenous job destruction on labour market volatility. To set the stage, we review the key equations in a continuous time search and matching model based on Mortensen and Pissarides (1994). There are two types of shocks in the economy. Apart from the aggregate productivity shock, an individual firm may also receive an idiosyncratic productivity shock at

Poisson rate  $\tau$ .<sup>27</sup> The productivity of a job is now written as  $yx$ , where  $y$  denotes, as in basic matching model, a general productivity parameter and  $x$  an idiosyncratic one. When an idiosyncratic shock arrives,  $x$  moves from its initial value to some new value  $x'$  drawing from a general distribution  $G(x)$  with support in the range  $0 \leq x \leq 1$ . The firm chooses a reservation productivity  $R$ , defined as the productivity level below which production is not profitable, and destroys jobs whose productivity falls below it. We assume any newly established firm-worker pair has value of  $x$  equal to 1.

The value of a job with productivity in the range  $R \leq x \leq 1$  satisfies

$$rJ(x) = yx - w(x) + \tau \int_R^1 J(s) dG(s) - \tau J(x) \quad (5.1)$$

The value of a job to a worker satisfies

$$rW(x) = w(x) + \tau \int_R^1 W(s) dG(s) + \tau G(R)U - \tau W(x) \quad (5.2)$$

The value of being unemployed is same as the model with exogenous job separation, we write it in a continuous time form as

$$rU = b + \theta q(\theta)[W(1) - U] \quad (5.3)$$

The value of vacancy to the firm is written as<sup>28</sup>

$$rV = -yc + q(\theta)[J(1) - V] \quad (5.4)$$

Free entry in the labour market drives rents from vacant jobs to zero. Therefore the equilibrium condition for vacancy creation is  $V = 0$ , implying that

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<sup>27</sup>In the basic matching model,  $\tau$  is the exogenous separation rate. Adapting to this change, we now write the model in continuous time.

<sup>28</sup>Following Pissarides (2000), we assume the vacancy posting cost is proportional to the productivity.



$$J(1) = \frac{yc}{q(\theta)} \quad (5.5)$$

We assume that the wage is determined by the Nash sharing rule<sup>29</sup>

$$w(x) = (1 - \phi)b + \phi y(x + c\theta) \quad (5.6)$$

Substitute the wage equation into (5.1) giving

$$rJ(x) = yx - (1 - \phi)b - \phi y(x + c\theta) + \tau \int_R^1 J(s) dG(s) - \tau J(x) \quad (5.7)$$

Next, evaluate (5.7) at  $x = R$  and subtract the result from (5.7),

$$(r + \tau)J(x) = (1 - \phi)y(x - R) \quad (5.8)$$

Evaluating (5.8) at  $x = 1$  and making use of zero-profit condition, we have the job creation condition

$$(1 - \phi) \frac{1 - R}{r + \tau} = \frac{c}{q(\theta)} \quad (5.9)$$

The job creation condition states that the expected gain from a new job must be equal to the expected hiring cost that the firm has to pay.

Substitute (5.8) into the integral expression of (5.7) and evaluate the expression at  $x = R$ , we obtain the job destruction condition satisfies

$$R - \frac{b}{y} - \frac{\phi c}{1 - \phi} \theta + \frac{\tau}{r + \tau} \int_R^1 (s - R) dG(s) = 0 \quad (5.10)$$

Expression (5.10) says the reservation productivity plus a positive option value (the integral part) which can be obtained once a new idiosyncratic shock arrives should be equal to the reservation wage ( $yR + \frac{y\tau}{r+\tau} \int_R^1 (s - R) dG(s) = rU = b - \frac{\phi}{1-\phi} cy\theta$ ). Since the reservation productivity is less than the reservation wage, there is some labour hoarding in the sense that

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<sup>29</sup>For the details of the derivation, see chapter 3.

firm keeps some currently unprofitable workers.

The evolution of unemployment is given by

$$\dot{u} = \tau G(R)(1 - u) - \theta q(\theta)u \quad (5.11)$$

Therefore the equilibrium unemployment is

$$u = \frac{\tau G(R)}{\tau G(R) + \theta q(\theta)} \quad (5.12)$$

### 5.1 Reservation Productivity and Labour Market Volatility

The job creation condition and job destruction condition jointly determine reservation productivity and labour market tightness. With knowledge of  $R$  and  $\theta$ , unemployment is pinned down by the Beveridge curve (5.12). By combining the job creation condition and job destruction condition, we can find an equilibrium condition for labour market tightness.

**Lemma 9** *Labour market tightness can be obtained by solving the following equilibrium condition*

$$\frac{r}{r + \tau} \left[ 1 - \frac{c(r + \tau)}{(1 - \phi)q(\theta)} \right] - \frac{\tau}{r + \tau} \int_{1 - \frac{c(r + \tau)}{(1 - \phi)q(\theta)}}^1 G(s) ds - \frac{\phi c}{1 - \phi} \theta - \frac{b}{y} + \frac{\tau}{r + \tau} = 0 \quad (5.13)$$

**Proof.** Rewrite the integral expression of the job destruction condition as follows,

$$\int_R^1 (s - R) dG(s) = \int_R^1 (s - R) f(s) ds = \int_R^1 s f(s) ds - \int_R^1 R f(s) ds$$

Using integration by parts to solve  $\int_R^1 s f(s) ds$ , we obtain

$$\begin{aligned} \int_R^1 (s - R) dG(s) &= sG(s) \Big|_R^1 - \int_R^1 G(s) ds - RG(s) \Big|_R^1 \\ &= 1 - RG(R) - R + RG(R) - \int_R^1 G(s) ds = 1 - R - \int_R^1 G(s) ds \end{aligned}$$

Substitute this into the job destruction condition, we obtain

$$R - \frac{b}{y} - \frac{\phi c}{1 - \phi} \theta + \frac{\tau}{r + \tau} (1 - R - \int_R^1 G(s) ds) = 0 \quad (5.14)$$

Rearrange (5.14),

$$\frac{r}{r + \tau} R - \frac{\tau}{r + \tau} \int_R^1 G(s) ds - \frac{\phi c}{1 - \phi} \theta - \frac{b}{y} + \frac{\tau}{r + \tau} = 0 \quad (5.15)$$

Derive the reservation productivity from the job creation condition,

$$R = 1 - \frac{c(r + \tau)}{(1 - \phi)q(\theta)} \quad (5.16)$$

Once we substitute (5.16) into (5.15), we get (5.13). ■

After implicit differentiation of (5.13), we have the following proposition.

**Proposition 10** *Given a reservation productivity  $R$ , the elasticity of labour market tightness  $\theta$  with respect to the productivity  $y$  is*

$$\eta_{\theta,y} = \frac{(r + \tau)}{\alpha[r + \tau G(R)] + \phi \theta q(\theta)} \frac{1}{1 - R} \frac{b}{y}. \quad (5.17)$$

**Proof.** Implicit differentiation of (5.13) yields

$$F_\theta = \frac{c[r + \tau G(R)]}{(1 - \phi)q^2(\theta)} \frac{\partial q(\theta)}{\partial \theta} - \frac{\phi c}{1 - \phi}, F_y = \frac{b}{y^2}$$

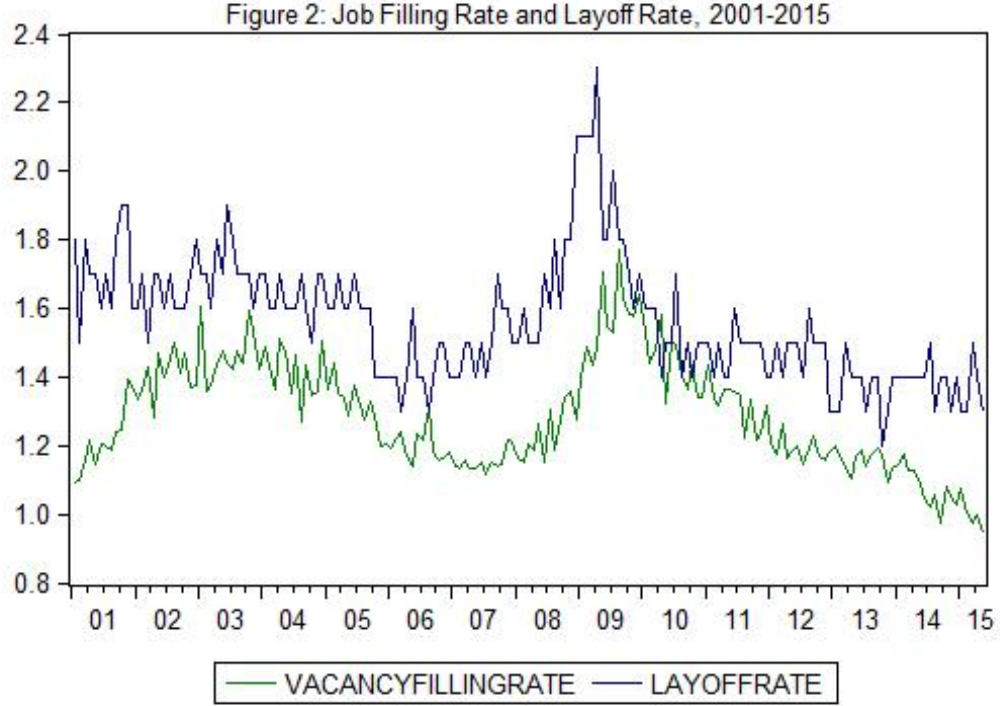
Substitute  $F_\theta$  and  $F_y$  into  $\eta_{\theta,y}$ ,

$$\eta_{\theta,y} = -\frac{F_y}{F_\theta} \frac{y}{\theta} = -\frac{\frac{b}{y^2}}{\frac{c[r + \tau G(R)]}{(1 - \phi)q^2(\theta)} \frac{\partial q(\theta)}{\partial \theta} - \frac{\phi c}{1 - \phi}} \frac{y}{\theta}$$

since we know that  $\theta q'(\theta)/q(\theta) = -\alpha$ , substitute this into the above equation,

$$\eta_{\theta,y} = \frac{\frac{b}{y}}{\frac{c[r + \tau G(R)]}{(1 - \phi)q(\theta)} \alpha + \frac{\phi c \theta}{1 - \phi}} = \frac{(1 - \phi)bq(\theta)}{\alpha[r + \tau G(R)]cy + \phi cy \theta q(\theta)}$$

Substitute  $R = 1 - \frac{c(r + \tau)}{(1 - \phi)q(\theta)}$  into this, we have (5.17) ■



The reservation productivity  $R$  enters two multiplicative terms of the elasticity of labour market tightness. In the first term, the role of  $R$  is limited since  $\alpha\tau$  is much smaller than  $\theta q(\theta)$  under reasonable calibrations. Therefore it is the second term in (5.17) that is critical in generating movements in labour market tightness. A large  $R$  will generate a large value of the second term. A large  $R$  also moves to  $G(R)$  closer to unity, which decreases the first term. Since the second term dominates, we have the following corollary:

**Corollary 11** *An increase in reservation productivity will increase the volatility of labour market tightness.*

A high reservation productivity increases layoffs. This drives up the flows into unemployment. As a result, the flows out of unemployment also

increases. This is because more new firms will enter the labour market and open job vacancies. The increase in both flows generate larger fluctuations in unemployment.

Equation (5.16) shows that one cause of a high reservation productivity is a slack labour market since then the vacancy filling rate is high. This ensures an employer can more easily fill a newly created more productive job to replace the current less productive one. A positive relation between the vacancy filling rate and the reservation productivity is also reflected in U.S data. Figure 2 displays the similar movements in job filling rate and layoff rate<sup>30</sup>.

From proposition 10 we know that labour market tightness is positively correlated with the aggregate productivity. Low aggregate productivity therefore is associated with low labour market tightness, a high vacancy filling rate and a large reservation productivity. So in a recession the reservation productivity will increase. This leads to more layoffs, therefore increasing the flows into unemployment. This further reduces labour market tightness and encourages new hires. However as aggregate productivity is low, inflows to unemployment are larger than outflows. So unemployment increases. The larger flows into and out of unemployment generate larger unemployment volatility. This implies

**Corollary 12** *A decrease in aggregate productivity leads to unemployment and unemployment volatility both to increase.*

**Proof.** According to proposition 10, we know that  $\partial\theta/\partial y > 0$ . So a decrease in aggregate productivity leads to a decrease in labour market tightness. This leads to an increase in the vacancy filling rate (see e.q. 2.6). From (5.16), we know this will increase the reservation productivity. Then according to corollary 11, the volatility of labour market tightness will increase. So unemployment volatility will increase.

Since a decrease in aggregate productivity leads to a decrease in labour market tightness, the job finding rate will also decrease. We already prove

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<sup>30</sup>To obtain the job filling rate, we divide the hires by job openings. We use the monthly data on layoff rate, hires and job openings from JOLTS website.

that when  $y$  decreases the reservation productivity will increase so  $G(R)$  will increase. According to the Beveridge curve (5.12), the decrease in  $\theta q(\theta)$  and the increase in  $G(R)$  will lead to unemployment to increase. ■

## 5.2 Discounts Rates and Reservation Productivity

Under the conventional view, the reservation productivity depends on the risk-free discounts rate. In a financial crisis, risk-free discounts rates drop so reservation productivity increases. This leads to more job separations and therefore larger flows into unemployment. Following Hall (2015), if we assume that  $r$  is the discount rate in the stock market, one implication is that the reservation productivity should have decreased in the recent recession since the high risk premium increased the discounts rate. This implies less job separations and therefore less flows into unemployment during the recession. This contradicts the facts.

## 6 Conclusion

This chapter gives a critical commentary on recent developments on the literature of solving unemployment volatility puzzle. Given the findings in this chapter, we have a few suggestions for future study on labour market volatility.

If say the evidence favours sticky wages in the sense of insulation of the wage from labour productivity, this chapter shows that the strategic wage bargaining cannot generate enough wage stickiness to replicate the observed labour market volatility if a more plausible assumption on the cost of delay is adopted. This calls for other approaches to model wage stickiness.

The literature has noted the co-movement between U.S labour and financial market but hasn't found a plausible channel to explain how the fluctuations in financial market are transmitted to the labour market. Our analysis on Hall (2015) shows that in the absence of capital and investment, purely assuming the discount rates in the stock market as the linkage is not

convincing.

Our discussion on endogenous job destruction indicates that job separation may be an important factor for labour market volatility especially in recessions. Future research should focus on the causes of a high reservation production and explore what policies can reduce this threshold.

## APPENDIX OF CHAPTER 4

### Discussions on the Cyclicalities of Vacancy Cost

Pissarides (2009) sheds light on the role of hiring costs in solving the unemployment volatility puzzle. The paper argues that apart from the conventional hiring cost,  $c$ , which is proportional to the duration of vacancies, there is another component of hiring costs that is independent of the duration of vacancies. If the hiring costs are shifted from the proportional to the fixed component, the volatility of job creation increases. Surprisingly, Pissarides (2009) and all other literature in this field neglects one question: should the hiring cost  $c$  be fixed or proportional to the productivity? This appendix argues that the cyclicality of  $c$  is crucial for solving the unemployment volatility puzzle. A pro-cyclical  $c$  can make the cost of hiring a worker even more pro-cyclical, which reduces the response of hiring to the productivity shock.

#### **A constant vacancy cost**

If vacancy cost are constant and equal to  $c$ , then, after imposing the zero profit condition  $V = 0$ , equation (2.4) implies

$$J = \frac{c}{\beta q(\theta)}$$

which we can substitute into equation (2.3) to arrive at

$$w = y - \frac{r + \tau}{q(\theta)} c \quad (\text{I})$$

Nash bargaining implies

$$w = b + \phi(y - b + \theta c) \quad (\text{II})$$

The two expressions for the wage jointly determine the equilibrium value of  $\theta$ :

$$y - b = \frac{r + \tau + \phi \theta q(\theta)}{(1 - \phi) q(\theta)} c \quad (\text{III})$$



After implicit differentiation of expression (10), we can compute the elasticity of market tightness with respect to productivity as

$$\eta_{\theta,y} = \frac{(r + \tau) + \phi\theta q(\theta)}{\alpha(r + \tau) + \phi\theta q(\theta)} \frac{y}{y - b} \quad (\text{IV})$$

### Cyclical vacancy costs

Next, assume instead that the vacancy costs are  $cy$ ; since the mean of productivity is unity, these alternatives are equivalent in steady-state.

Equilibrium condition (III) can be re-written as

$$y - b = \frac{r + \tau + \phi\theta q(\theta)}{(1 - \phi)q(\theta)} cy \quad (\text{V})$$

The elasticity of market tightness with respect to productivity now becomes as

$$\eta_{\theta,y} = \frac{(1 - \phi)q(\theta)}{\alpha(r + \tau) + \phi\theta q(\theta)} \frac{b}{cy} \quad (\text{VI})$$

Substituting the expression (V) into (VI) to replace  $cy$  gives

$$\eta_{\theta,y} = \frac{r + \tau + \phi\theta q(\theta)}{\alpha(r + \tau) + \phi\theta q(\theta)} \frac{b}{y - b} \quad (\text{VII})$$

### Discussion

Comparing (IV) with (VII), the first term in both expressions is the same, however the second term  $\frac{y}{y-b}$  in expression (IV) is replaced by  $\frac{b}{y-b}$  in (VII). Under the standard calibration, value of leisure is smaller than the output, so  $\eta_{\theta,y}$  is smaller when  $c$  is pro-cyclical.

**Proposition 13** *In a labour market with search frictions, an individual firm pays  $c$  to post a vacancy and pays  $w$  for wage. The marginal benefit of a hire is  $\lambda = y - w$ . If the cost of posting a vacancy is pro-cyclical, we have  $\frac{\partial w}{\partial y} |_{cy} > \frac{\partial w}{\partial y} |_c$  and  $\frac{\partial \lambda}{\partial y} |_{cy} < \frac{\partial \lambda}{\partial y} |_c$ .*

**Proof.** Recall the wage equation in case of a constant cost of posting a vacancy,  $w = (1 - \phi)b + \phi(y + \theta c)$ , take the derivative of the wage with

respect to the productivity,

$$\frac{\partial w}{\partial y} \big|_c = \phi + \phi c \frac{\partial \theta}{\partial y}$$

Similarly, take the derivative to the wage equation in case of a cyclical vacancy cost,  $w = (1 - \phi)b + \phi y(1 + \theta c)$ ,

$$\frac{\partial w}{\partial y} \big|_{cy} = \phi(1 + \theta c) + \phi cy \frac{\partial \theta}{\partial y}$$

Since  $cy = c$  holds in equilibrium, this also implies that the two cases share the same equilibrium. Therefore, we have

$$\frac{\partial w}{\partial y} \big|_{cy} - \frac{\partial w}{\partial y} \big|_c = \phi \theta c > 0$$

Given the fact that  $1 = \frac{\partial w}{\partial y} + \frac{\partial \lambda}{\partial y}$ , we also have  $\frac{\partial \lambda}{\partial y} \big|_{cy} < \frac{\partial \lambda}{\partial y} \big|_c$ . ■

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